Quantum Mechanics I

Problem Set 5

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Exercise 32

Suppose that $A|\psi\rangle = a|\psi\rangle$. Show that

i) $f(A)|\psi\rangle = f(a)|\psi\rangle$,

ii) $\langle\psi|f(A)^\dagger = f(a)^*\langle\psi|$.

Exercise 33

Show that if $[A, B] = 0$ (A and B commute), then A and B can be simultaneously diagonalized (there exists an orthonormal basis of common eigenvectors.)

Exercise 34

(See discussion in class.) Imagine that initially our two-flavor system has flavor f_e: $F|\psi(t_0)\rangle = f_e|\psi(t_0)\rangle$.

i) What is the probability that the system has the same flavor at time $t = t_0 + \Delta t$?

ii) What is the probability that the flavor is f_μ at time $t_0 + \Delta t$?

iii) Plot both probabilities as a function of Δt.

Exercise 35

The one-dimensional delta function $\delta(x)$ is defined by the relation

$$\int_{-\infty}^{\infty} f(x)\delta(x) = f(0).$$

i) Show that $\int f(x)\delta(x-a) = f(a)$.

ii) Show that $\delta(x)$ can be represented by the limit

$$\delta(x) = \lim_{\sigma\to 0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right).$$

iii) Make a "plot" of the delta function.

iv) What is $d\delta/dx$?

v) What is $\delta(f(x))$?

vi) What is the delta function (in Cartesian coordinates) in n dimensions?

vii) What is $\delta^2(x)$?