IX. AMPERE'S LAW

INTRODUCTION

Two integral laws describing properties of magnetic field are:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, I$$

The first equation says that magnetic field flux through any closed surface is always zero. According to Gauss' Law electric field flux through a closed surface is equal to the net electric charge inside the surface. Thus, we can interpret the first equation here as stating that there are no magnetic charges. In fact, magnetic fields are created by electric charges in motion i.e. electric current.

The second equation, known as Ampere's Law, states that integral of magnetic field along any closed path is directly proportional to the net electric current crossing any surface bounded by the closed path. Utility of Ampere's Law for calculation of magnetic fields is similar to the applications of Gauss' Law for calculations of electric fields.

PURPOSE

Experimental verification of Ampere's Law.

PRE-LAB ASSIGNMENTS

A. Readings:

To illustrate Ampere's Law let us consider $\oint \vec{B} \cdot d\vec{s}$ for the magnetic field created by a current loop along the closed path indicated by the dashed line in Fig. 1. To carry out this integral we will move along the path in counter-clockwise direction. We need to sum up length along the path multiplied by magnetic field component tangent to the path, $B_{\|}$, since $\vec{B} \cdot d\vec{s} = B_{\|}\, ds$. If vector $\vec{B}_{\|}$ has the same direction as vector $d\vec{s}$ we consider $B_{\|}$ positive (e.g. point 1 in Fig. 1). If vector $\vec{B}_{\|}$ has the opposite direction to vector $d\vec{s}$ we consider $B_{\|}$ negative (e.g. point 2 in Fig. 1). There is no electric current passing through the area bounded by the dashed line, therefore, according to Ampere's Law this sum should be zero. In other words, all positive contributions to the sum must be canceled by negative contributions.

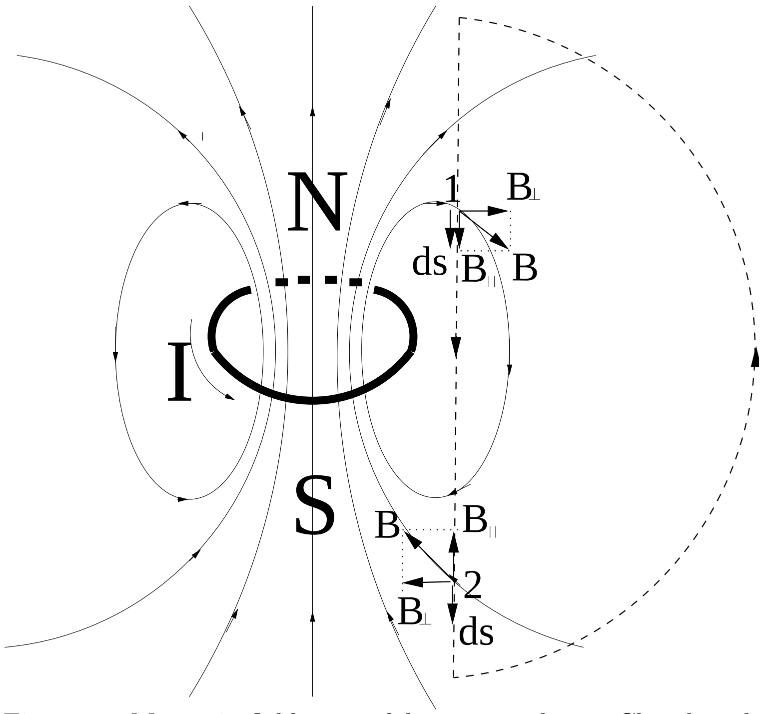


Figure 1. Magnetic field created by current loop. Closed path integral is indicated by the dashed line. Decomposition of the magnetic field vector ($\vec{B}$) into parallel ($\vec{B}_{\|}$) and perpendicular ($\vec{B}_{\perp}$) components to the path interval $d\vec{s}$ is shown at two different points. Ampere's integral is a sum of $B_{\|}\, ds$ over the entire closed path.

Let us now choose a different closed path for the same magnetic field. If we make the path to be as depicted in Fig. 2, then the current in the wire loop will pass the area bounded by the path and the Ampere's integral will be different from zero and equal to $\mu_0\, I$.

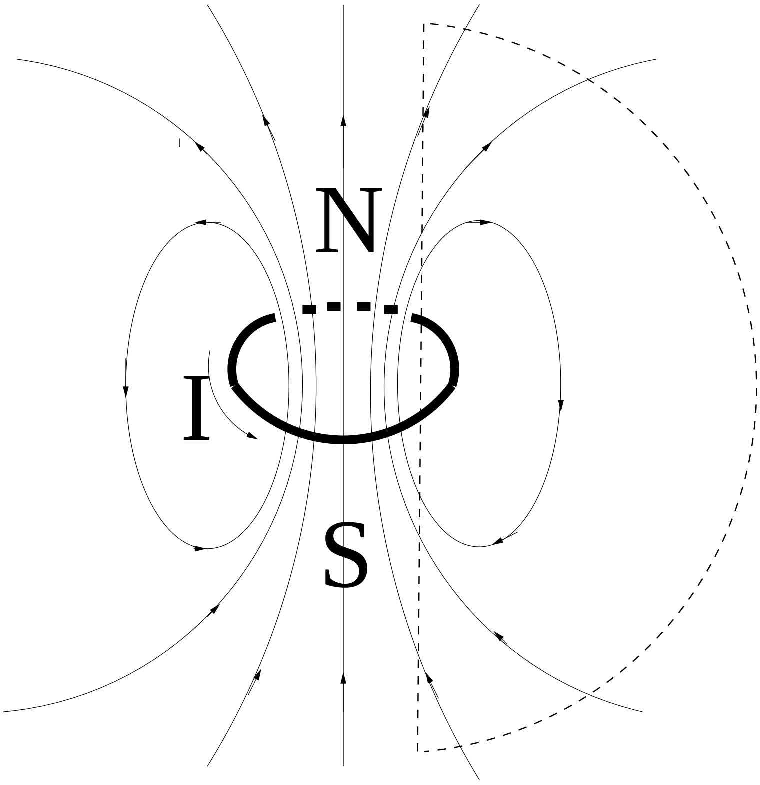


Figure 2. Different choice of the closed path.

You may have wandered about particular shape of the closed path used in the examples above. This shape has been chosen because it is related to how we will do measurements. The closed path integral can be split into the integral along the straight section and along the half-circle: $\oint = \int_{straight\ line} + \int_{half\ circle}$. If the straight line is made sufficiently long, then the circular part will fall into the region of space in which magnetic field is very weak. This can be seen in Fig. 1 and 2 since the half-circle goes through the region with smaller field line density as the straight section. If B is very small along the half-circle part then $\int_{half\ circle} \approx 0$ and $\oint \approx \int_{straight\ line}$. Therefore, it is enough to measure magnetic field along the straight section to obtain the field integral over the closed path.

B. Exercises:

Please answer the questions on Report Sheet IX–1, which will be collected at the *beginning* of the laboratory session and graded by your instructor.

REPORT SHEET IX–1

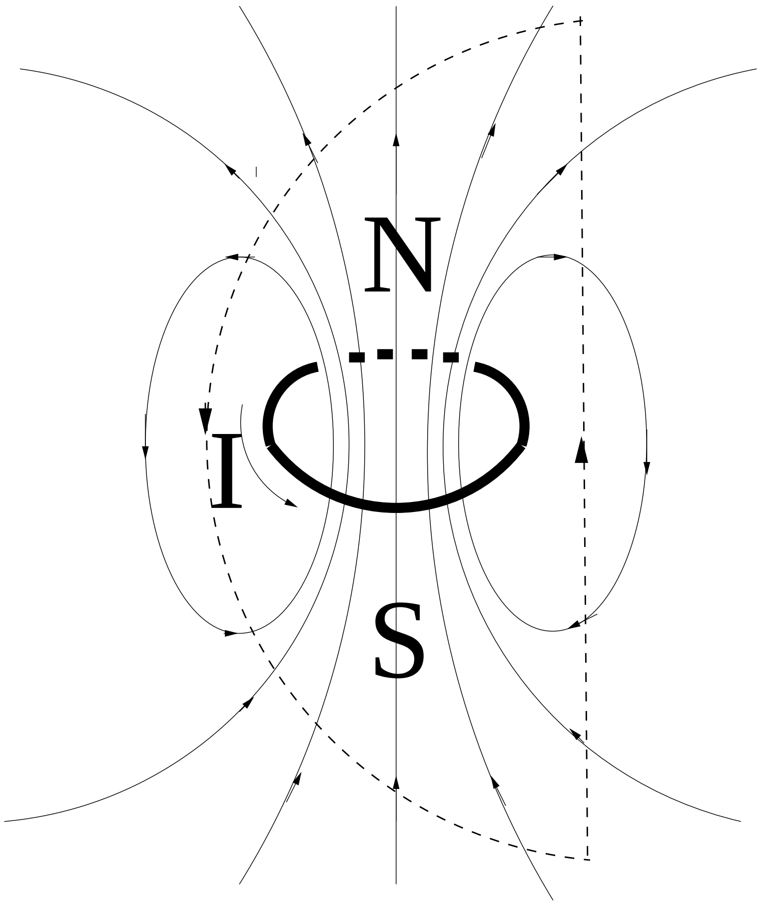
Date________________________________ Name________________________________

Instructor________________________________

PRE-LAB EXERCISES

What does Ampere's Law say about $\oint \vec{B} \cdot d\vec{s}$ along the closed path indicated by the dashed line drawn below?

Hint: When adding currents crossing a surface you need to pay attention to relative direction of the currents with respect to each other.



$$\oint \vec{B} \cdot d\vec{s} =$$

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LABORATORY ASSIGNMENTS

Materials Needed:

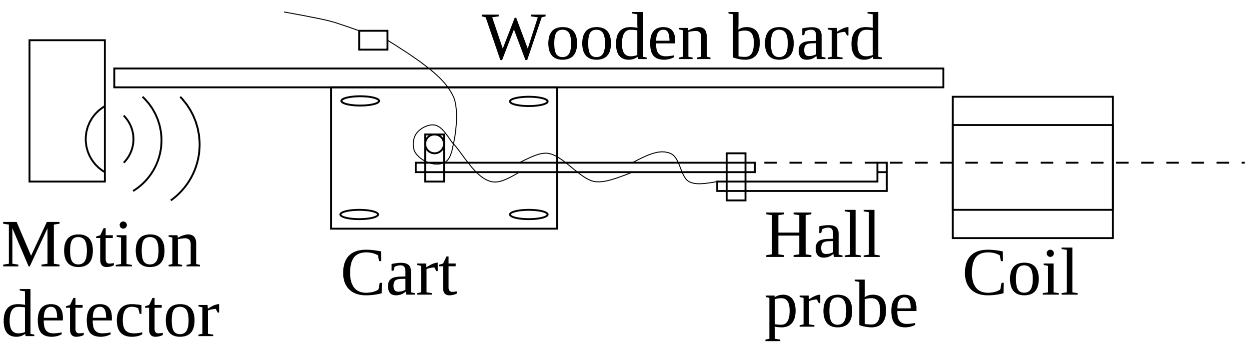
- Solenoid
- Power Supply
- 220Ω Resistor
- Permanent Magnet: large one and a bunch of small ones
- Hall Probe
- Motion Detector
- ULI computer interface box
- Cart
- Mounting fixtures
- Wooden guiding block
- Cables
- Voltmeter

Experiment A-C: *Verification of Ampere's Law*

The Task:

To graph magnetic field variation with position along the line crossing the field. To calculate closed-path integral of the magnetic field and to compare with the predictions of Ampere's Law.

Experiment A: *Verification of Ampere's Law for Solenoidal Magnetic Field*



Procedures

A-1. Mount the Hall probe on a cart with help of long wooden stick. Use long wooden board to guide the cart. Position solenoid at one end of the board and the motion detector at the other end as illustrated above. Adjust the mounting of the probe and the position of the solenoid such that the probe tip will cross the center of solenoid when the cart is moved. The probe orientation must be parallel to the direction of motion. Wrap the probe output cables around the mounting stick and the rod so they do not obstruct motion of the cart. You will be moving the cart from the initial position at about 0.5m away from the Motion Detector until the cart reaches the solenoid. The probe will move from one side of solenoid, through its center, to the other side. At the initial and final positions the probe tip must be at some distance away from the solenoid in the region in which magnetic field is small. Make sure you can move the probe through the coil without any obstructions. You may have to adjust mounting fixtures to achieve this.

A-2. Connect the Hall probe to DIN1 and the motion detector to PORT2 of the ULI interface box. The probe should be set to low sensitivity. Switch the interface box on. Start the computer, and click on the PHY222 icon to start the program. To load the proper initialization file, choose "Open…" from the "File" menu. Open the file "ampere" in PHY222 subdirectory. There are four graphs in this set-up. The upper left graph will display position of the cart as a function of time. The bottom left plot will show the magnetic field as a function of time. The upper right plot will show the magnetic field as a function of the cart (i.e. probe) position.

With the power supply off and all permanent magnets far away from the probe go to "Experiment" menu and select "Zero", then "Zero magnetic field" from the pop-out window.

Connect the solenoid to the power supply together with 220Ω resistor connected in series to reduce voltage supplied to the solenoid.

A-3. Switch the power supply on and turn the knob to the maximum setting.

Determine the closest distance of the cart to the motion detector in which the position is still measured. Mark this position. This will be the initial position of the cart for all measurements.

Collect the data while moving the cart from the initial position until it reaches the solenoid. Make sure that nothing obstructs the space in between the motion detector and the cart (like your hand). If the Distance vs. Time graph shows some discontinuities you make have to reposition the motion detector. If nothing helps try attaching some flat screen to the end of the cart so it can be easier detected by the motion detector. Make also sure that the probe does not saturate when going through the solenoid center (this would be seen as flat part in the Magnetic Field vs. Distance graph on top of the distribution). Lower voltage on the power supply if you do observe saturation.

Once you are satisfied with your measurement copy Magnetic Field vs. Distance graph onto the left graph in Report Sheet IX–2. For all graphs in your report indicate minimal and maximal values of the field detected. Also draw a horizontal line which corresponds to magnetic field equal to zero. The area under the curve you see on this graph[1] is equal to to $\oint \vec{B} \cdot d\vec{s}$ appearing in Ampere's Law. To integrate the area under the Magnetic Field vs. Distance graph select it by click on it (somewhere in the middle

[1]If the curve goes below zero we count the area as negative.

of it). Then go to "Analyze" menu and click on "Integral". The value of the integral should be displayed on the graph. The integrated area is highlighted. If the highlighted area does not cover your entire distance range you need to select the integration range by stretching it on the graph with a mouse. Indicate value of the integral on Report Sheet IX–2 below the graph.

A-4. Adjust the probe mounting such that it will pass through the solenoid close to its winding rather than at its center. Repeat the measurement as in the previous step. Copy Magnetic Field vs. Distance graph onto the right graph in Report Sheet IX–2, integrate it and report the value under the graph. Answer questions related to the integrals you have measured.

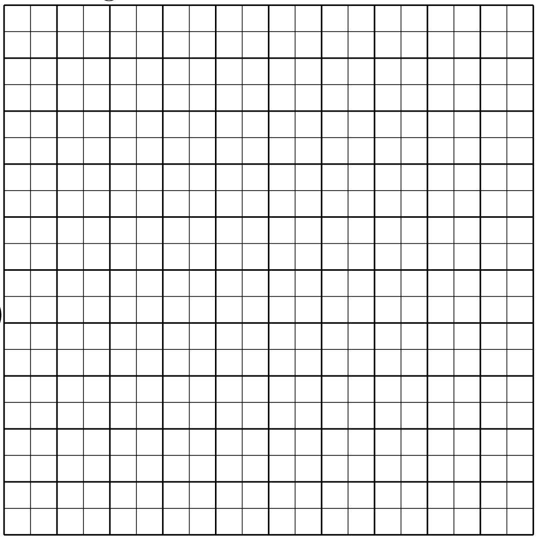
REPORT SHEET IX–2

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

A-3. Magnetic Field vs. Distance

$B_{\|}$ (Gauss)

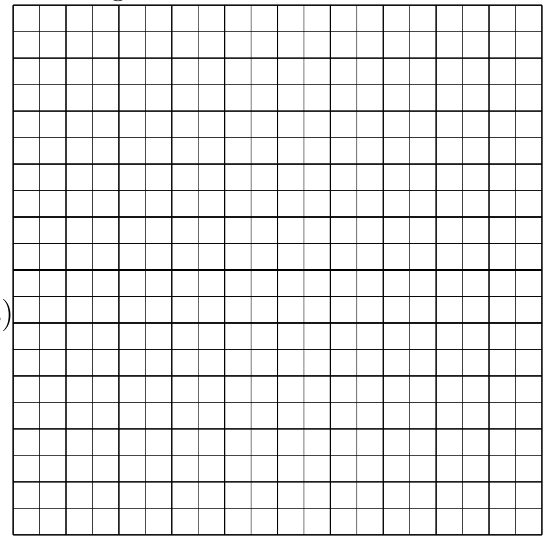


Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

A-4. Magnetic Field vs. Distance

$B_{\|}$ (Gauss)



Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

A-4.

From Ampere's Law are the two field integrals expected to be:

☐ the same ☐ different

Do your measurements roughly confirm this expectation?

☐ yes ☐ no

A-5.

$V =$ (V)

$R =$ (Ω)

$I = V/R =$ (A)

$N =$

$\mu_0 = 4\pi\, 10^{-3}$ (Gauss·m/A)

$\mu_0\, N\, I =$ (Gauss·m)

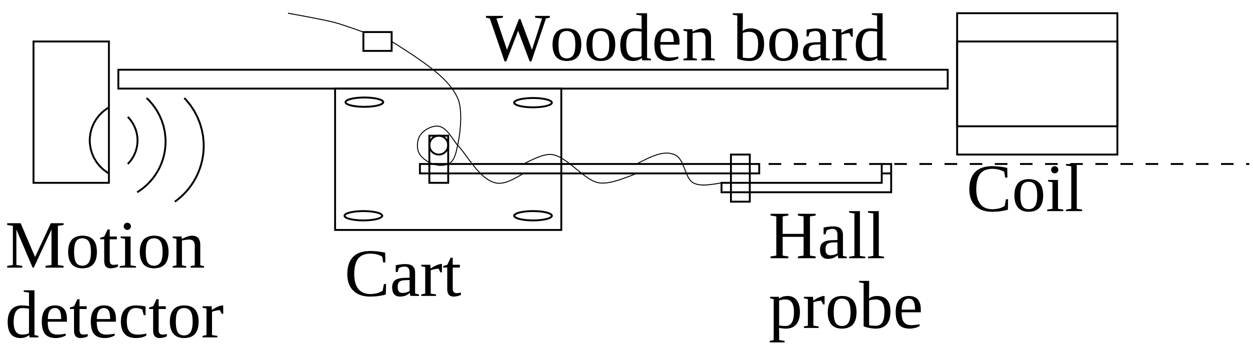
Do your measurements roughly agree with the expected value of the integral?

☐ yes ☐ no

blank

A-5. In this step we will verify the Ampere's law quantitatively. You should have observed that magnetic field drops essentially to zero at the beginning and at the end of your straight path through the solenoid. If you this is not what you observe you may need to zero your probe again (see A-2) or extend the probe mounting on the cart so it can move farther away from the solenoid in its final position. Imagine that we close the measurement path by moving the probe along half-circle (like in Fig. 2) maintaining large distance to the solenoid. The magnetic field will remain small, thus this part of the path will not add substantially to the field integral. Amount of current crossing area of our closed path is $N \times I$, where N is the number of turns of the solenoid wire and I is the electric current flowing through the wire. N is given on the solenoid label. To determine I measure potential drop across the solenoid (excluding the resistor) with hand-held voltmeter. Read resistance of the solenoid from its label and calculate I using Ohm's Law. Calculate value of the field integral predicted by the Ampere's Law ($\mu_0 N I$) and compare it to the measured values.

A-6. Now measure field integral along the path just outside the solenoid. Move the solenoid to the position shown below. Try to get as close to the solenoid with your probe as you can. Report your measurements in Report Sheet IX–3. Answer the related questions.



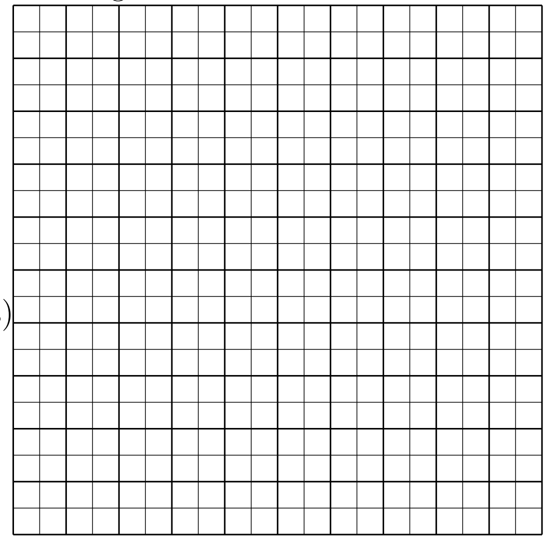
REPORT SHEET IX–3

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

A-6. Magnetic Field vs. Distance

$B_{\|}$ (Gauss)



Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

A-6.

From Ampere's Law is the field integral here expected to be:

☐ the same as in A-3,4 ☐ zero

Do your measurements roughly confirm this expectation?

☐ yes ☐ no

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Experiment B: *Verification of Ampere's Law for Permanent Magnets*

Procedures

B-1. Switch the power supply off. Put a large permanent magnet in place of the solenoid directly under the the line of the probe motion. Make N-S line of the permanent magnet vertical (i.e. the magnet will rest on flat bottom). Collect the data while moving the probe through the magnetic field of the permanent magnet. You are likely to observe probe saturation (usually saturation value is different for positive and nagative fields). Rise the probe to avoid the saturation and take the data again Use Report Sheet IX–4 to report your measurements.

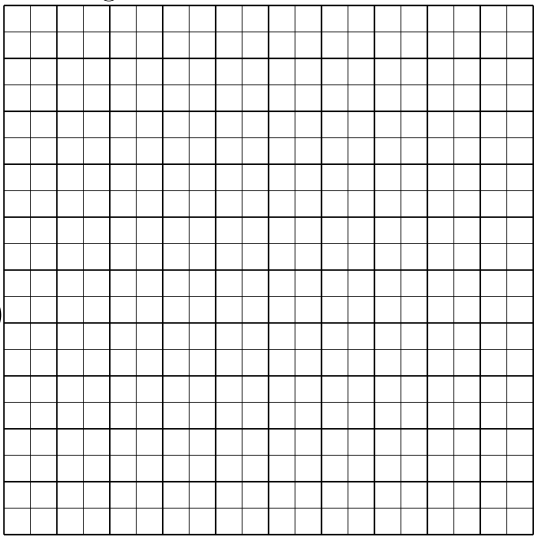
REPORT SHEET IX–4

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

B-1. Magnetic Field vs. Distance

$B_{\parallel}$ (Gauss)

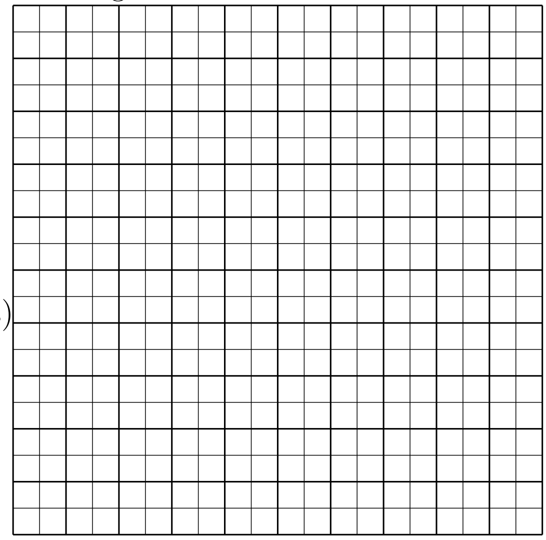


Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

B-2. Magnetic Field vs. Distance

$B_{\parallel}$ (Gauss)

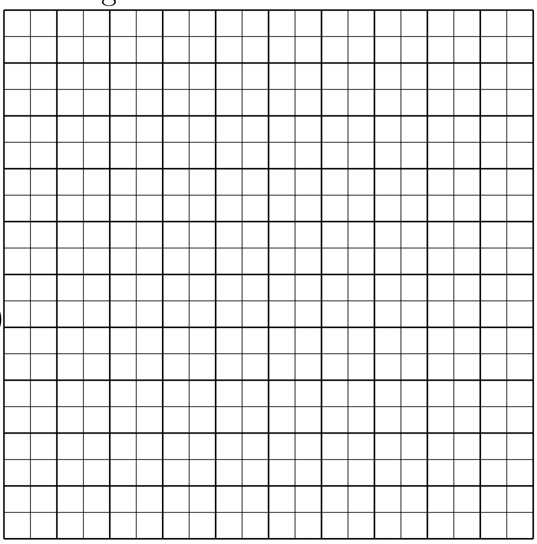


Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

B-3. Magnetic Field vs. Distance

$B_{\parallel}$ (Gauss)

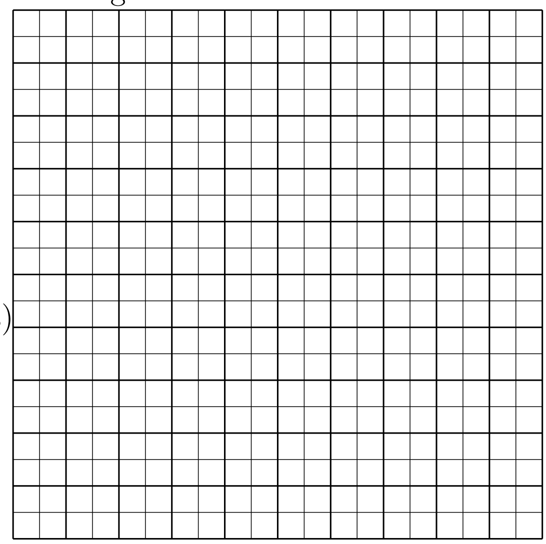


Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

B-4. Magnetic Field vs. Distance

$B_{\parallel}$ (Gauss)



Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

blank

B-2. Flip the magnet upside-down and repeat the measurement.

B-3. Now rest the magnet on its side, with N-S line in horizontal position. Repeat the measurement.

B-4. Again flip the magnet by 180 degrees. Repeat the measurement. Answer the questions in Report Sheet IX–5.

B-5. Move the probe above "mine field" of small permanent magnets. Use your imagination while positioning the magnets. You should lower the probe to get much closer to the magnets than in the previous measurements. Report your measurements in Report Sheet IX–5.

REPORT SHEET IX–5

Date______________________________ Name______________________________

Instructor______________________________ Partner(s)______________________________

B-4.

What is expected for the magnetic field integrals measured in B-1,B-2,B-3 and B-4 from Ampere's Law? (**Hint:** Think of permanent magnet as a small solenoid)

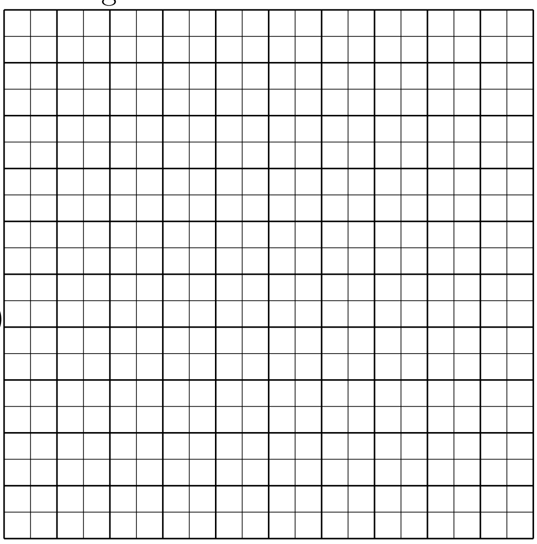
[ ] to be large [ ] to be zero

Do your measurements roughly confirm this expectation? [ ] yes [ ] no

What happens to the magnetic field when you flip the magnet by 180 degrees (compare B-1 with B-2 and B-3 with B-4)?

Try to relate pattern of the magnetic field changes with distance observed in B-1 and B-3 with the magnetic field sketched in Fig. 1 (the permanent magnet replaces the current loop in our case). Explain in words or sketch a picture for each situations with the field line directions and the measurement path (use the other side of this page if needed).

B-5.



$\oint \vec{B} \cdot d\vec{s} =$ $(Gauss \cdot m)$

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Experiment C: *Verification of Ampere's Law for the Combined Field of Solenoid and Permanent Magnets*

Procedures

C-1. Move the solenoid back to its initial position with the probe going through its center. Switch the power supply on and turn the knob to the same position as in Experiment A. Position the large and small permanent magnets near the solenoid. You can put small magnets inside the solenoid if you wish. Measure the integral of magnetic field. If the probe saturates you will need to reposition the magnets or the probe. Use Report Sheet IX–6 to report your measurements.

C-2. Switch the power supply off and quit the program before leaving the laboratory (don't save anything when quitting).

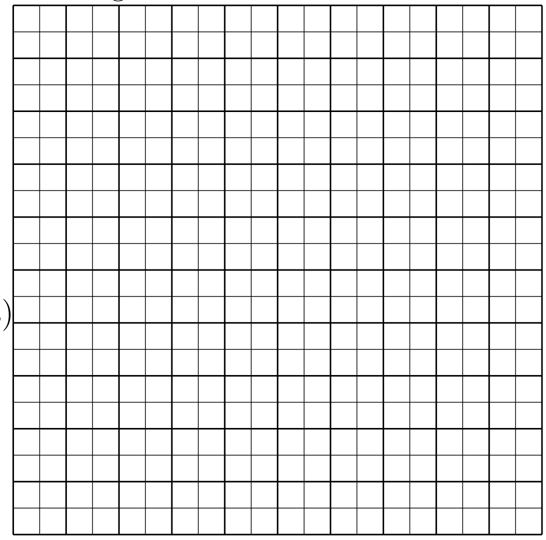
REPORT SHEET IX–6

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

C-1. Magnetic Field vs. Distance

$B_{\parallel}$ (Gauss)



Distance (m)

$\oint \vec{B}\cdot d\vec{s} =$ $(Gauss\cdot m)$

C-1.

From Ampere's Law is the field integral here expected to be:

☐ the same as in A-3,4 ☐ zero

Do your measurements roughly confirm this expectation?

☐ yes ☐ no