

INTRODUCTION

DC stands for “direct-current”. A direct current stays the same in magnitude and does not change its direction. Ohm’s Law and the rules for effective resistance of resistors connected in series and in parallel, which we studied in the previous experiment, can be used to understand only simple DC circuits. More general approach, which can be applied to analyze any DC circuit, is based on so called Kirchhoff’s Rules.

PURPOSE

- Experimental verification of Kirchhoff’s Rules.
- Analysis of a complex DC circuit with Kirchhoff’s Rules.

PRE-LAB ASSIGNMENTS

A. Readings:

There are two Kirchhoff's Rules:

junction rule - The sum of the currents entering any junction must equal the sum of the currents leaving that junction.

loop rule - The sum of the potential differences across each element around any closed circuit loop must be zero.

The first rule reflects more general **law of charge conservation**. (Net charge cannot disappear or pop-out from nothing at the junction). The second rule stems from the **law of energy conservation**. Since the electric forces are conservative, energy done by electric forces on a test charge along any closed loop must be zero.

Let us apply Kirchhoff's Rules to the circuit shown in Fig. 1. In fact, we will study this circuit experimentally as well. This is an example of the circuit which cannot be understood from the rules for resistors in series and in parallel.

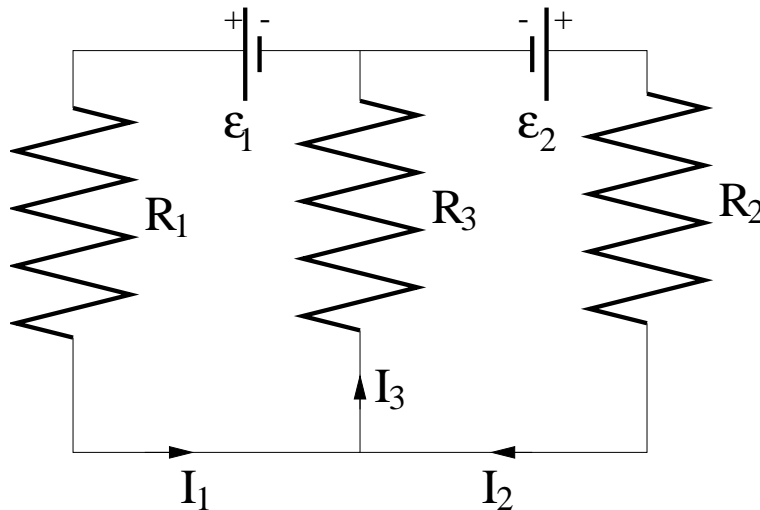


Figure 1. DC circuit analyzed in Experiment C.

In the first step current direction in each loop needs to be assumed. It does not matter if we assume a wrong direction, since the sign of the current in the solution to the Kirchhoff's Rules will tell us if we assumed the right direction (negative solution indicates the opposite current direction). Fig. 1 shows one possible direction of currents that we adopt here.

There are two junctions in our circuit. In general, for N junctions it is enough to apply the junction rule to $N - 1$ junctions. Thus, we need to write the first rule only once:

$$I_1 + I_2 = I_3 \quad (1)$$

There are two loops in our circuit. We need to apply the second rule to each of them. Moving around the loops in the direction of the assumed current flow we have:

$$I_1 R_1 + I_3 R_3 - \epsilon_1 = 0 \quad (\text{loop on the left}) \quad (2)$$

$$I_2 R_2 + I_3 R_3 - \epsilon_2 = 0 \quad (\text{loop on the right}) \quad (3)$$

For the potential difference across a resistor, we have used Ohm's law ($V_i = I_i R_i$).

Assuming that the resistances and the potentials supplied by the batteries are known, we need to solve the equations above for three currents, I_1 , I_2 and I_3 . Since the number of equations equals the number of unknowns, the problem can be solved. By eliminating I_3 (putting (1) into (2) and (3)) we can reduce the problem to two equations with two unknowns (I_1 and I_2).

$$\begin{cases} I_1 R_1 + (I_1 + I_2) R_3 = \epsilon_1 \\ I_2 R_2 + (I_1 + I_2) R_3 = \epsilon_2 \end{cases} \quad (4)$$

In the next step we can use the first equation above to express I_2 in terms of I_1 and put this into the second equation. This will leave us with one equation with one unknown (I_1). Solving it for I_1 we get:

$$I_1 = \frac{\epsilon_1 - \epsilon_2 \frac{R_3}{R_2 + R_3}}{R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}} \quad (5)$$

Because of the symmetry of the circuit we can obtain formula for I_2 by swapping indices 1 and 2. Finally, I_3 can be obtained from the sum of I_1 and I_2 .

B. Exercises:

Please answer the questions on Report Sheet V-1, which will be collected at the *beginning* of the laboratory session and graded by your instructor.

REPORT SHEET V-1

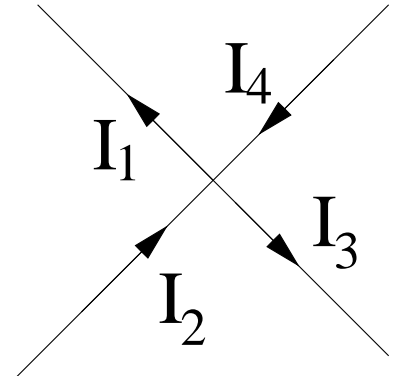
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PRE-LAB EXERCISES

Exercise 1.

Write junction rule for the junction shown below:



Exercise 2.

Assume that in the circuit shown in Fig. 1 all resistances are the same, $R_1 = R_2 = R_3 = R$. Express I_1 in terms of ϵ_1 , ϵ_2 and R . You can start from equation (5). Show your algebra.

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LABORATORY ASSIGNMENTS

Experiment A: Junction Rule

The Task:

To experimentally verify Kirchhoff's Junction Rule.

Materials Needed:

- Resistors: $10k\Omega$, two 220Ω
- Rheostat
- Power Supply
- Dual Channel Amplifier with two voltage probes
- ULI computer interface box
- 4.5V Battery (for apparatus tests only)
- Voltmeter (for apparatus tests only)
- Cables

Procedures

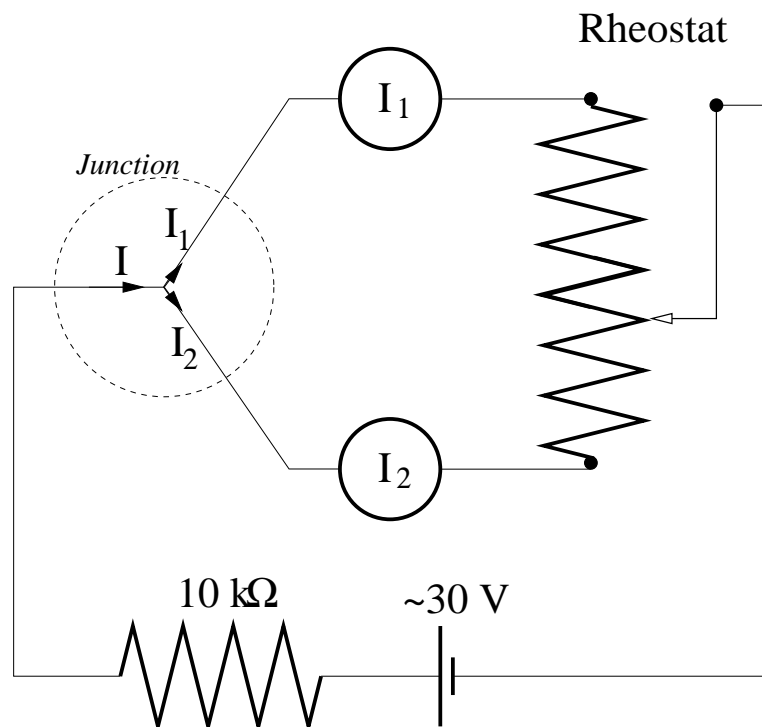


Figure 2. DC circuit used to verify Kirchhoff's Junction Rule.

A-1. The circuit used in this experiment is shown in Fig. 2. Pattern of current flow in the junction will be varied by moving the slider of the rheostat which changes resistance of both circuit branches. Ideally we should measure I , I_1 , I_2 and check if

$$I = I_1 + I_2 \quad (6)$$

Since we can measure only two currents at the same time, we will measure currents leaving the junction I_1 and I_2 and we will keep the current entering the junction, I , constant. Then, it is enough to check that $I_1 + I_2$ remains constant while I_1 and I_2 vary.

Instead of a battery we will use DC power supply which provides constant potential difference around 30V. Changing position of the slider changes the effective resistance of the entire circuit, thus changing I . However, $10k\Omega$ resistor in our circuit has resistance much larger than any other element of the circuit, thus dominating the effective resistance of entire circuit. Therefore, the effective resistance and the current I will be approximately constant.

To measure currents I_1 and I_2 we will measure potential drops over two 220Ω resistors ($I_i = V_i/220$). Connect the circuit as shown in Fig. 2. Connect 220Ω resistors at the places indicated by the current meters.

A-2 We will use two voltage probes in this experiment. Make sure the probes are connected to the Dual Channel Amplifier and that the latter is connected to the ULI interface box (DIN1 to DIN1, DIN2 to DIN2). Switch the interface box on. Start the computer, and click on the PHY222 icon to start the program. To load the proper initialization file, choose “Open...” from the “File” menu. Open the file “dc-rule1” in PHY222 subdirectory.

Check the voltage probes using a test battery and hand-held voltmeter.

A-3 There are four graphs on your screen. The top (bottom) left graph shows I_1 (I_2) as a function of time. The top right graph shows dependence of I_1 on I_2 . Finally, the bottom right plot shows dependence of the sum of the currents $I_1 + I_2$ on time.

Switch the power supply on and set it to the maximal potential difference. Make both I_1 and I_2 positive (reverse the order in which the voltage probe leads are connected to the 220Ω resistor if you need to change the sign).

Click on “Collect” and move the slider of the rheostat in its full range. Copy I_1 vs. I_2 and $I_1 + I_2$ vs. Time graphs onto Report Sheet V-2. Did the junction rule work?

A-4 Determine mean value of $I_1 + I_2$ from your data (click on $I_1 + I_2$ vs. Time graph, “Analyze” menu \rightarrow “Statistics”) and compare it to the value of I expected from Ohm’s Law (Report Sheet V-2). To calculate the expected value of I assume that the effective resistance of the entire circuit is approximately $10k\Omega$ (since this is an underestimate for the total resistance, we are likely to slightly overestimate I). Make sure that you use the same units for $I_1 + I_2$ and I in this comparison.

REPORT SHEET V-2

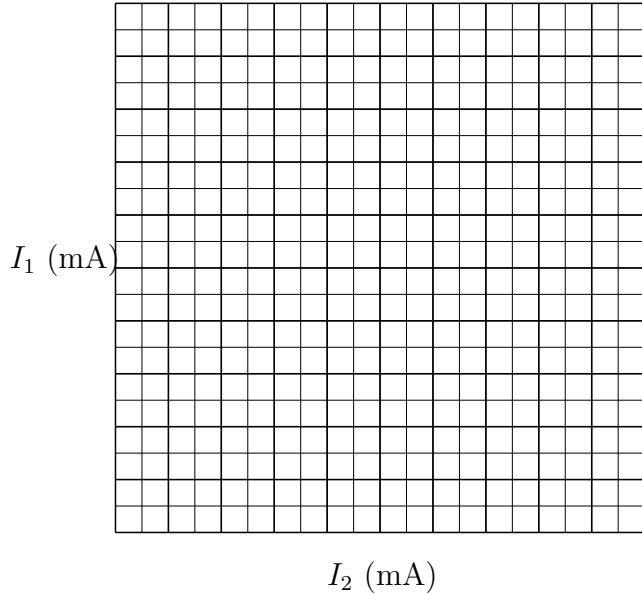
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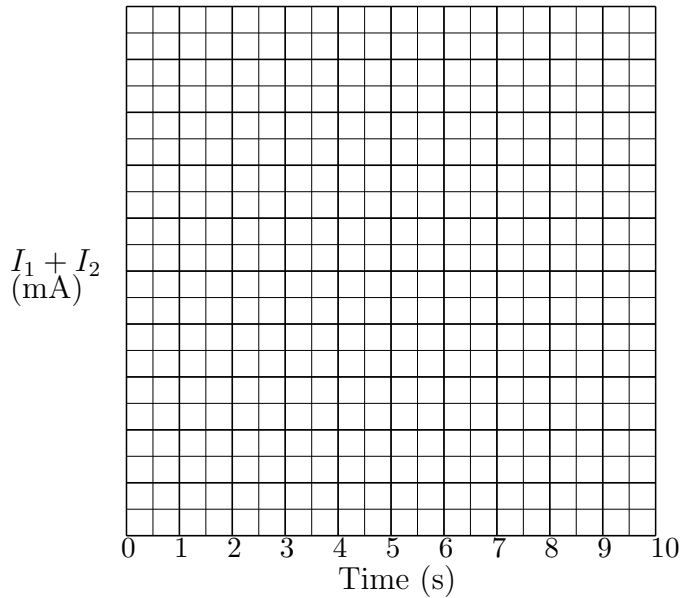
A-3. Current 1 vs. Current 2



A-3

Are your $I_1 + I_2$ vs. Time data consistent with Kirchhoff's Junction Rule? Explain.

A-3. Current1+Current2 vs. Time



A-4

Expected value of I from Ohm's Law

$$I = \epsilon/R \approx$$

Mean value of

$$I_1 + I_2 =$$

Do these values roughly confirm the junction rule: $I = I_1 + I_2$?

blank

Experiment B: Loop Rule

The Task:

To experimentally verify Kirchhoff's Loop Rule.

Materials Needed:

- Resistor: 220Ω
- Rheostat
- Dual Channel Amplifier with two voltage probes
- ULI computer interface box
- 4.5V Battery
- Cables

Procedures

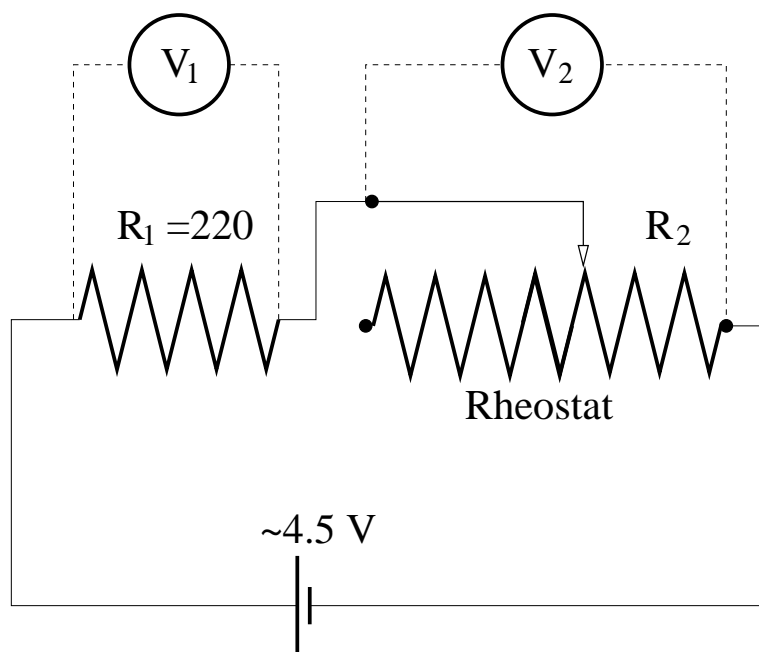


Figure 3. DC circuit used to verify Kirchhoff's Loop Rule.

B-1. The circuit used in this experiment is shown in Fig. 3. There are three elements in this loop: battery and two resistors (rheostat plays a role of one of them). Kirchhoff's second rule written for this loop is:

$$V_1 + V_2 - \epsilon = 0 \quad (7)$$

where V_1 is the potential difference across the first resistor ($R_1 = 220\Omega$), V_2 is the potential difference across the second resistor (i.e. rheostat - R_2 is variable) and ϵ is the potential difference generated by the battery ($\epsilon \approx 4.5V$). Since ϵ is constant, to

prove the second rule we need to demonstrate that $V_1 + V_2$ remains constant while V_1 and V_2 can vary individually.

Go to “File” menu and “Open...” to load “dc-rule2” set-up file. When the program asks you if you would like to save changes to the previous set-up click on “No”.

Connect the circuit as shown in Fig. 3. Make sure that one connection to the rheostat is to the slider. Connect one voltage probe across 220Ω resistor and the second one across the rheostat. Make both voltage readings positive (reverse the order of the voltage lead connections if necessary).

- B-2.** Click on “Collect” and move the slider of the rheostat in its full range. Copy V_1 vs. V_2 and $V_1 + V_2$ vs. Time graphs onto Report Sheet V-3. Did the loop rule work?
- B-3** Determine mean value of $V_1 + V_2$ from your data and compare it to the value of ϵ measured for the battery (Report Sheet V-3).

REPORT SHEET V-3

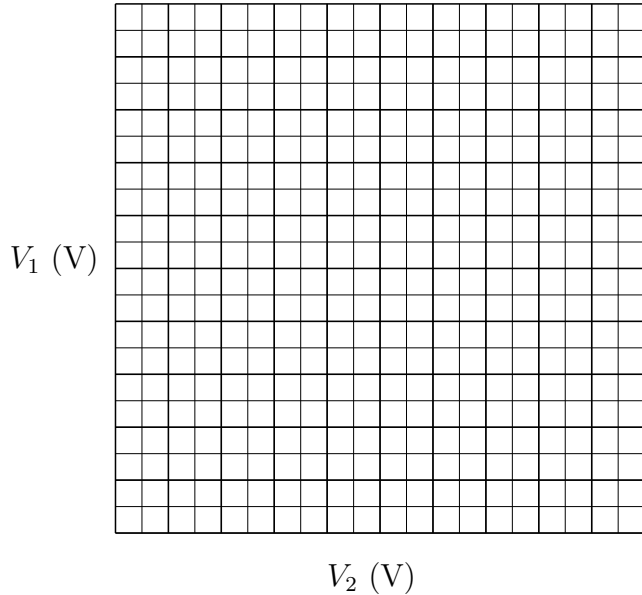
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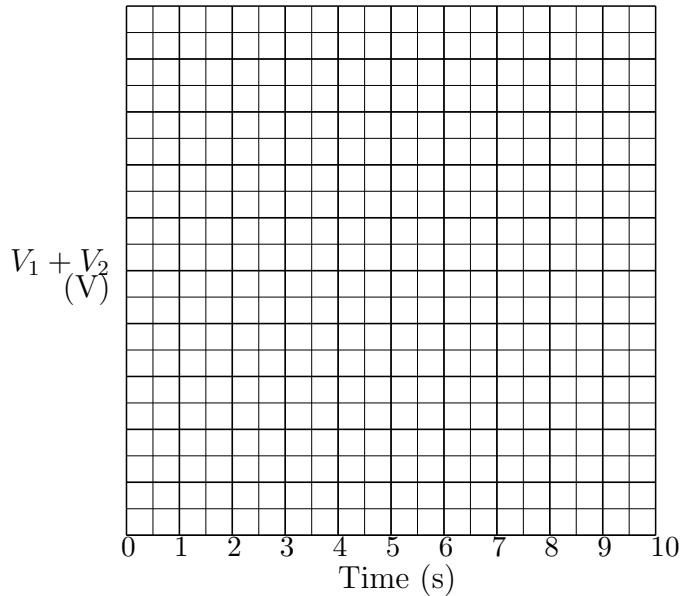
B-2. Potential 1 vs. Potential 2



B-2

Are your $V_1 + V_2$ vs. Time data consistent with Kirchhoff's Loop Rule? Explain.

B-2. Potential1+Potential2 vs. Time



B-3

Potential difference generated by the battery

$$\epsilon =$$

Mean value of

$$V_1 + V_2 =$$

Do these values confirm the loop rule:
 $\epsilon = V_1 + V_2?$

blank

Experiment C: Complex DC Circuit

The Task:

To experimentally verify predictions based on the solution to Kirchhoff's Junction and Loop Rules.

Materials Needed:

- Power Supply
- 4.5V Battery
- Voltage Divider Box
- Three 100Ω Resistors
- Dual Channel Amplifier with two voltage probes
- ULI computer interface box
- Cables

Procedures

We will now study circuit which served as an example in the theoretical introduction (Fig. 1). We will use $R_1 = R_2 = R_3 = R = 100\Omega$, 4.5V battery for ϵ_2 and the power supply as a source of variable ϵ_1 . Using the formula (5) derived in the introduction we get:

$$I_1 = \frac{2\epsilon_1 - \epsilon_2}{3R} \quad (8)$$

$$I_2 = \frac{2\epsilon_2 - \epsilon_1}{3R} \quad (9)$$

$$I_3 = I_1 + I_2 = \frac{\epsilon_1 + \epsilon_2}{3R} \quad (10)$$

To verify these equations we will measure dependence of the current (in each branch) on ϵ_1 .

C-1. Connect three 100Ω resistors as shown in Fig. 1. Connect 4.5V battery as ϵ_2 (pay attention to the polarity).

Connect the power supply across the terminals A and B of the voltage divider box (negative voltage to the terminal B). Connect the terminals B and C of the voltage divider box as ϵ_1 in the circuit. Pay attention to the polarity of the power supply connection; the negative pole of the power supply (black) should be on the side of negative pole of the battery.

To measure ϵ_1 connect the voltage probe 1 across the terminals C and B of the box. It is easy to make a mistake when wiring this circuit. Make sure that the voltage probe is **not** connected directly to the power supply (terminals A and B).

C-2 To verify formula (8) we will measure I_1 by potential difference reading across the resistor R_1 . Connect the second voltage probe across this resistor ($I_1 = V_2/100\Omega$).

Load the program set-up which was previously used to study Ohm's Law ("File" menu → "Open..." → file "ohms" in PHY222 subdirectory).

With the power supply on and set to the maximum, V_1 and V_2 (or $I \equiv I_1$) should be both positive. Switch the order of voltage probe lead connections to reverse the sign if needed.

Start collecting the data and vary $\epsilon_1 \equiv V_1$ by turning the knob of the power supply from its minimal to its maximal setting (you can also move back-and-forth between the extreme settings).

- C-3** The formula (8) predicts that the current I_1 should be zero (i.e. no current flowing through the first loop) for $\epsilon_1 = \epsilon_2/2$. From your I vs. V graph read-out V for which the current is zero. Is it half of the potential difference supplied by the battery? (see Report Sheet V-4).
- C-4** We can further verify formula (8) by fitting straight line to the Current vs. Potential data. Select this graph by clicking on it. Then go to "Analyze" menu and select "Linear Fit". From the box superimposed on the graph read the slope and the intercept of the fitted line ($y = mx + b$, $x = \epsilon_1$, $y = I_1$, m =slope, b =intercept). From formula (8) slope should be $\frac{2}{3R}$ and intercept $-\frac{\epsilon_2}{3R}$. Put into these formulae $R = 100\Omega$ and the battery voltage for ϵ_2 . To compare to the fitted slope and intercept multiply your theoretical predictions by 1000, since the current on your graph is displayed in units of mA rather than A . Report the measured and predicted values in Report Sheet V-4.
- C-5** Switch the voltage probe 2 to measure current I_2 . Collect data for varying ϵ_1 . Fit a line to your I vs. V graph. Compare the fitted slope and intercept to the expected ones from formula (9). Report the measured and predicted values in Report Sheet V-4.
- C-6** Switch the voltage probe 2 to measure current I_3 . Collect data to verify formula (10).

REPORT SHEET V-4

Date _____ Name _____

Instructor _____ Partner(s) _____

C-3. $I_1 = 0$ corresponds to $\epsilon_1 =$
Half of battery voltage $\epsilon_2/2 =$

	Slope			Intercept		
	Expected		Measured	Expected		Measured
	Formula	Value (mA/V)		Formula	Value (mA)	
C-4 I_1 vs. ϵ_1	$\frac{2}{3R}$			$-\frac{\epsilon_2}{3R}$		
C-5 I_2 vs. ϵ_1						
C-6 I_3 vs. ϵ_1						

