

## II. ELECTROSTATIC FIELDS --- --- s01.01

### INTRODUCTION

Modern physics describes forces in terms of fields. Electric and magnetic fields are central concepts of the classical theory of electromagnetism. Instead of specifying formulae for the forces between charged bodies at rest, interactions among these bodies can be described by use of electrostatic fields. Every charged body creates an electrostatic field in its surroundings. Conversely, every charged body placed in such a field experiences an electrostatic force.

### PURPOSE

In this experiment you will study electrostatic fields created around charged objects of different shapes. The electrostatic fields will be represented graphically by use of equipotential lines and lines of force. You will become familiar with the concept of electric potential.

## PRE-LAB ASSIGNMENTS

### A. Readings:

Many physics phenomena can be described in terms of a field. We are talking about a *field* when some value of a property can be assigned to every point in space. Electrostatic forces experienced by an electric charge in the vicinity of another electrically charged body can be described using the *electric field*. With each point in space around the charged body, we can associate a force experienced by a unit electric charge of positive sign:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The electric charge  $q_0$  (“test charge”) must be small enough to have a negligible influence on the electric field at the tested point. This is a vector quantity, since force is a vector. This force per unit charge is called the *electric field strength*,  $\vec{E}$ . The actual force experienced by an electric charge  $q$  at the point with electric field strength  $\vec{E}$  can be obtained from the equation:

$$\vec{F} = q\vec{E}.$$

For visualization purposes, it is convenient to introduce *lines of force*. The lines of force are tangent to  $\vec{E}$  at every point, and are drawn so that the number of lines per unit cross-section area is proportional to the magnitude of the electric field strength,  $E$ . Thus,  $E$  is large where the lines are dense and small where they are far apart. The electric field can be described not only by the *vector* field of  $\vec{E}$ , but also by the scalar field of *electric potential*. The electric potential difference between two points in space, A and B, is the work that must be done to move, without acceleration, a unit electric charge of positive sign from point A to point B:

$$V_B - V_A = \frac{W_{AB}}{q_0}.$$

At each point along the path between these two points, external forces must balance the electric force to prevent acceleration,  $\vec{F}_{ext} = -q_0\vec{E}$ . The work done by these forces at the straight path interval  $dr$  is  $W_{dr} = F_{ext}^{\parallel} dr$ , where  $F_{ext}^{\parallel}$  is the component of  $\vec{F}_{ext}$  parallel to the path orientation. This can also be written as  $W_{dr} = \vec{F}_{ext} \cdot d\vec{r} = -q_0 \vec{E} \cdot d\vec{r}$ . Any path between points A and B can be represented as a sum of many such small straight intervals. Thus  $W_{AB} = \sum_i W_{dr_i} = -q_0 \sum_i \vec{E}_i \cdot d\vec{r}_i$ , or more precisely,  $W_{AB} = -q_0 \int_A^B \vec{E} \cdot d\vec{r}$ . Therefore,

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{r}.$$

As a matter of convention it is assumed that the electric potential is zero at infinity:  $V_{\infty} = 0$ . Thus,

$$V_B = V_B - V_{\infty} = - \int_{\infty}^B \vec{E} \cdot d\vec{r}.$$

You may have noticed that  $q_0$  canceled in the calculation of the potential difference. Furthermore, it turns out that the above integral (and thus  $W_{AB}$ ) does not depend on the exact path taken; rather, it depends only on the starting and stopping points. Thus,  $V_B$  is a property of the point  $B$ . The electric field can be described either by the vector field  $\vec{E}$  or the scalar field  $V$ , and both descriptions are completely equivalent. To calculate  $\vec{E}$  from the potential  $V$  we must invert the above expression:  $\vec{E} = -\frac{dV}{d\vec{r}} = -(\hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z})$ , where  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  are unit vectors in the directions of the coordinate axes. When the potential depends only on one coordinate,  $V(x, y, z) = V(x)$ , this can be written in a scalar form  $E = -\frac{dV}{dx}$ , where  $dV$  is the potential difference between two ends of the very small interval  $dx$ . All points that have the same electric potential together create an *equipotential surface* (*equipotential line* in a two dimensional cross-section). Since the potential difference between any two points on the equipotential surface is zero, no work is done while moving the test charge on this surface. This is possible only if the surface is perpendicular at every point to the electric field vector ( $\vec{E} \cdot d\vec{r} = 0$  at any point). In other words, *equipotential surfaces are perpendicular to the lines of force*. In a figure, it is convenient to draw equipotential lines such that the same difference of potential exists between adjacent lines. If the density of such lines is large, the electric field strength is large as well. Conversely, low electric field strength results in large distances between adjacent equipotential lines.

Let us illustrate the above concepts using the electric field created by a point-like charge  $Q$ . According to Coulomb's law, the electrostatic force experienced by the test charge  $q_0$  placed at a distance  $r$  from the charge  $Q$  is  $F = \frac{q_0 Q}{r^2}$  (neglecting constant factors). Thus,  $E = \frac{F}{q_0} = \frac{Q}{r^2}$  and  $V = -\int E dr = -Q \int \frac{dr}{r^2} = \frac{Q}{r}$ . The electric field of a point-like charge is represented graphically in Fig. 1. The solid curves indicate the equipotential lines. These are circles because the potential depends only on the distance from the center. The density of the equipotential lines (i.e. number of lines per unit area) is large near the center (since  $\frac{1}{r}$  is the largest here), and decreases when moving away from the charge. Similarly, the electric field strength decreases quadratically with distance from the charge  $Q$ . The dashed lines represent the lines of force, which are directed radially. Thus, the lines of force are indeed perpendicular to the equipotential lines.

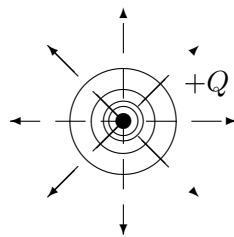


Figure 1. Electric field produced by a point-like charge. The circles represent the equipotential lines corresponding to a constant potential difference between any two adjacent lines. The dashed lines show a few lines of force.

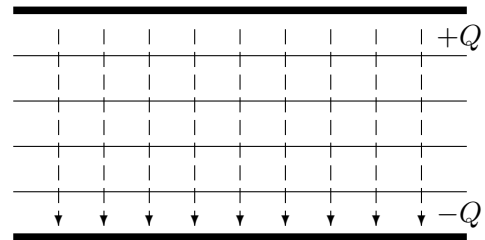


Figure 2. Electric field between two infinitely large flat surfaces parallel to each other and oppositely charged. The dashed lines of force represent the electric field, which is (always) perpendicular to the equipotential lines.

Figure 2 shows the electric field created between two infinitely large flat plates of opposite charge, parallel to each other. The field is *uniform*, i.e. the field strength is the same at every point. This uniformity is represented by parallel and equidistant equipotential lines.

**B. Exercises:**

Please answer the questions on Report Sheet II-1, which will be collected at the *beginning* of the laboratory session and graded by your instructor.

**REPORT SHEET II-1**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_

PRE-LAB EXERCISES

---

**Exercise 1.**

What is the definition of an equipotential line?

**Exercise 2.**

What is the definition of lines of force? What can you tell about relative orientation of lines of force and equipotential lines?

**Exercise 3.**

Gravitational forces create a field that is similar to electrostatic fields. (a) Draw a two-dimensional cross-section of the gravitational field as experienced by small objects close to the surface of the Earth. Indicate equipotential lines and lines of force. (b) Draw a two-dimensional cross-section of the gravitational field of the Earth on the scale of interplanetary distances.

(a)

(b)

blank

## LABORATORY ASSIGNMENTS

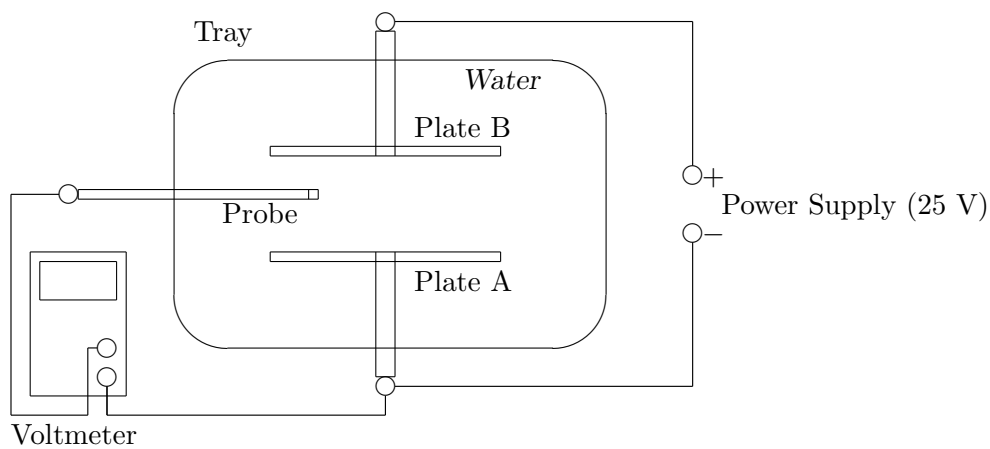


Figure 3. Diagram of the experimental set-up for determination of equipotential lines.

### **Experiment A–E:** *Determination Of Electrical Fields With The Electrolytic Tank Method*

#### **Materials Needed:**

- Tray with tap water
- Electrodes:
  - (A) Two flat plates, (B) 1/2 inch and 8 inch cylinders,
  - (C) Two 1/2 inch cylinders, (D) Three flat plates. (E) Any combination of these electrodes.
- Probe
- Voltmeter
- Power Supply
- Cables

#### **The Task:**

To trace equipotential lines between charged bodies of various shapes.

## Procedures

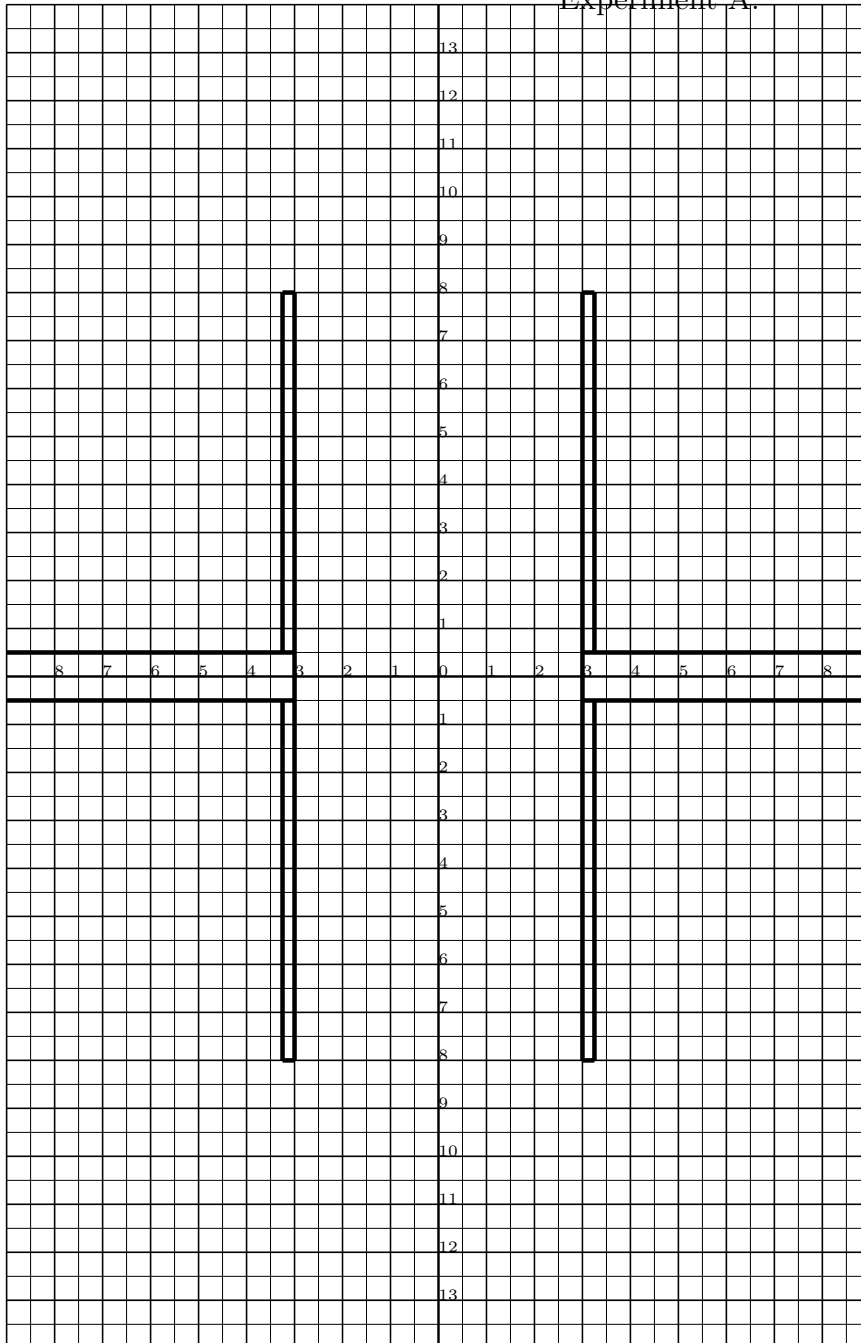
- A-1.** Set up the electrical circuit as shown in Fig. 3. If the circuit is already wired up, please check the connections and make sure that you understand the role of each element. **The power supply should be off until all connections are ready. Switch the power supply off whenever you change electrodes.** If by chance you provide a direct conductive path between the “+” and the “-” poles with the power supply on, it will trip. Make sure you remove the faulty connection before trying to reset the power supply. To reset the power supply switch it off and on again. If it still does not work, perhaps a fuse in the power supply has burned. Ask the instructor for help.
- A-2.** Fill the tray with tap water to a depth of about 1/4 inch.
- A-3.** Place the two flat electrodes parallel to each other and oriented along the length of the tray so that they are separated by a distance of 6 big blocks of the graph paper. Line the electrodes up very precisely on the graph paper, corresponding to the positions indicated on the Report Sheet II-2, and tape them in place on the side of the tray with the black adhesive tape.
- A-4.** Turn the voltage control knob on the power supply all the way to the left. Then, switch the power supply on.
- A-5.** Switch the voltmeter on and set it to read DC voltages. Touch with the probe to the plate B. Slowly turn the knob on the power supply to the right until you can read about 25 V on the voltmeter.
- A-6.** Now you are ready to trace equipotential lines. Put the probe into the tray and sample the potential for various locations of the probe. Trace the 20 V equipotential line from one end of the tray to the other, including the regions beyond the extent of the plates. Transfer coordinates of at least six points of this line to a 1/4 scale drawing on Report Sheet II-2. Connect the points with a smooth line.  
Repeat the procedure for the 15, 10 and 5 V equipotential lines. The lines should be labeled. At the end, using dotted lines, draw a few lines of force (should be perpendicular to the equipotential lines you found) in the whole region of the tray, not only between the electrodes. Answer the questions on the report sheet.

**REPORT SHEET II-2**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_ Partner(s) \_\_\_\_\_

Experiment A.



How does strength of electric field ( $E$ ) and its direction vary in the space between the electrodes? Justify your answer.

What can you say about the electric field in the space beyond the extent of the electrodes?

blank

- B.** Repeat the procedures necessary to trace out the 20, 15, 10 and 5 volt equipotential lines for the 1/2 inch and 8 inch concentric cylinders. Switch the power supply off when you exchange electrodes. Treat the large ring as the reference plate (plate *A*). Be careful to make them exactly concentric with one another, as shown on Report Sheet II-3. Such an arrangement corresponds to a coaxial transmission line of the type used in television, telephone and FM transmission. Make the drawing on Report Sheet II-3. Include the plots of the lines of force. Measure the potentials at the four points outside of the 8 inch cylinder indicated on the report sheet and answer the related question.

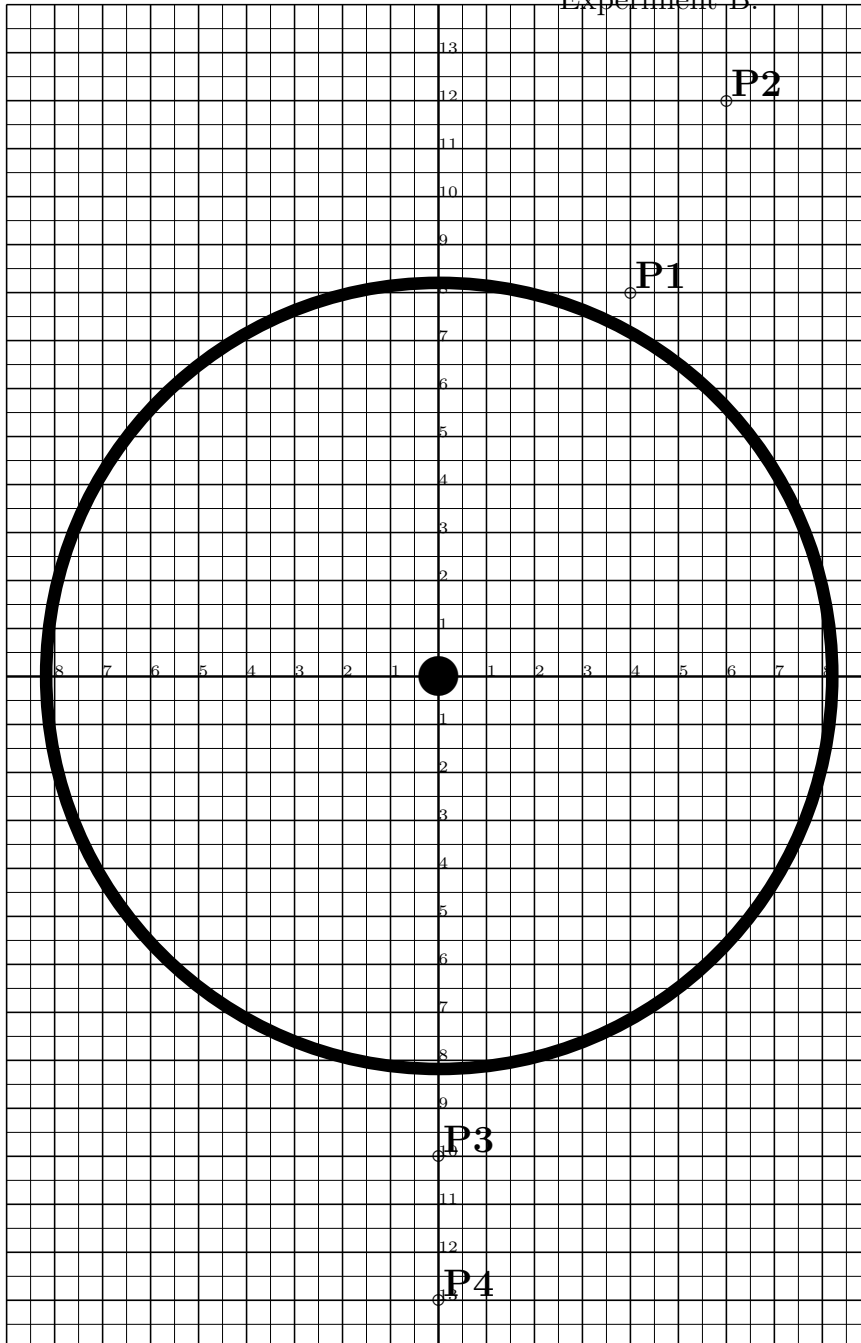


**REPORT SHEET II-3**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_ Partner(s) \_\_\_\_\_

Experiment B.



Value of the potential at the points outside the large cylinder:

$V_{P1} =$

$V_{P2} =$

$V_{P3} =$

$V_{P4} =$

How does the strength of electric field ( $E$ ) outside the large cylinder compare with the strength of electric field inside the cylinder?

blank

- C. Repeat the procedures necessary to trace out the 20, 15, 10, and 5 volt equipotential lines for two 1/2 inch round bodies with their centers 6 units (big squares) apart (Report Sheet II-4). Place the bodies symmetrically with respect to the long axis of the tray. Make the drawing of the equipotential lines and the lines of force on Report Sheet II-4.

*Note:* The field you are going to measure is equivalent to the two-dimensional cross-section of the field between two infinitely long wire conductors. Shade the region where the electric field is strongest.

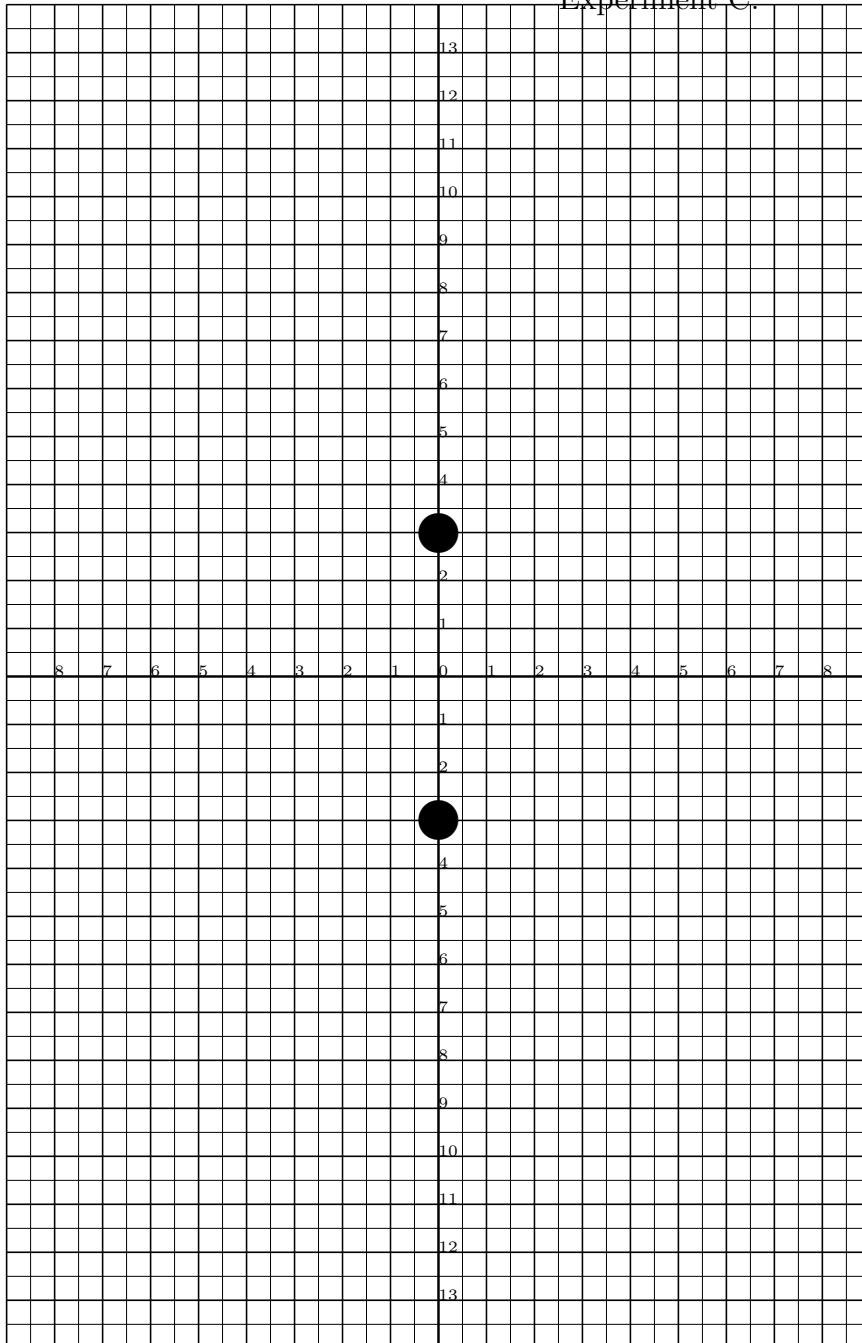


**REPORT SHEET II-4**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_ Partner(s) \_\_\_\_\_

Experiment C.



Shade the region where the electric field is strongest.

blank

- D.** Arrange the three flat plates in the tray as shown in the diagram on Report Sheet II-5. Leave 6 big squares from point  $C$  to the plate A. Repeat the procedures necessary to trace out the 20, 15, 10, and 5 volt equipotential lines for this configuration. Draw also the lines of force. Shade the region where the electric field is strongest.

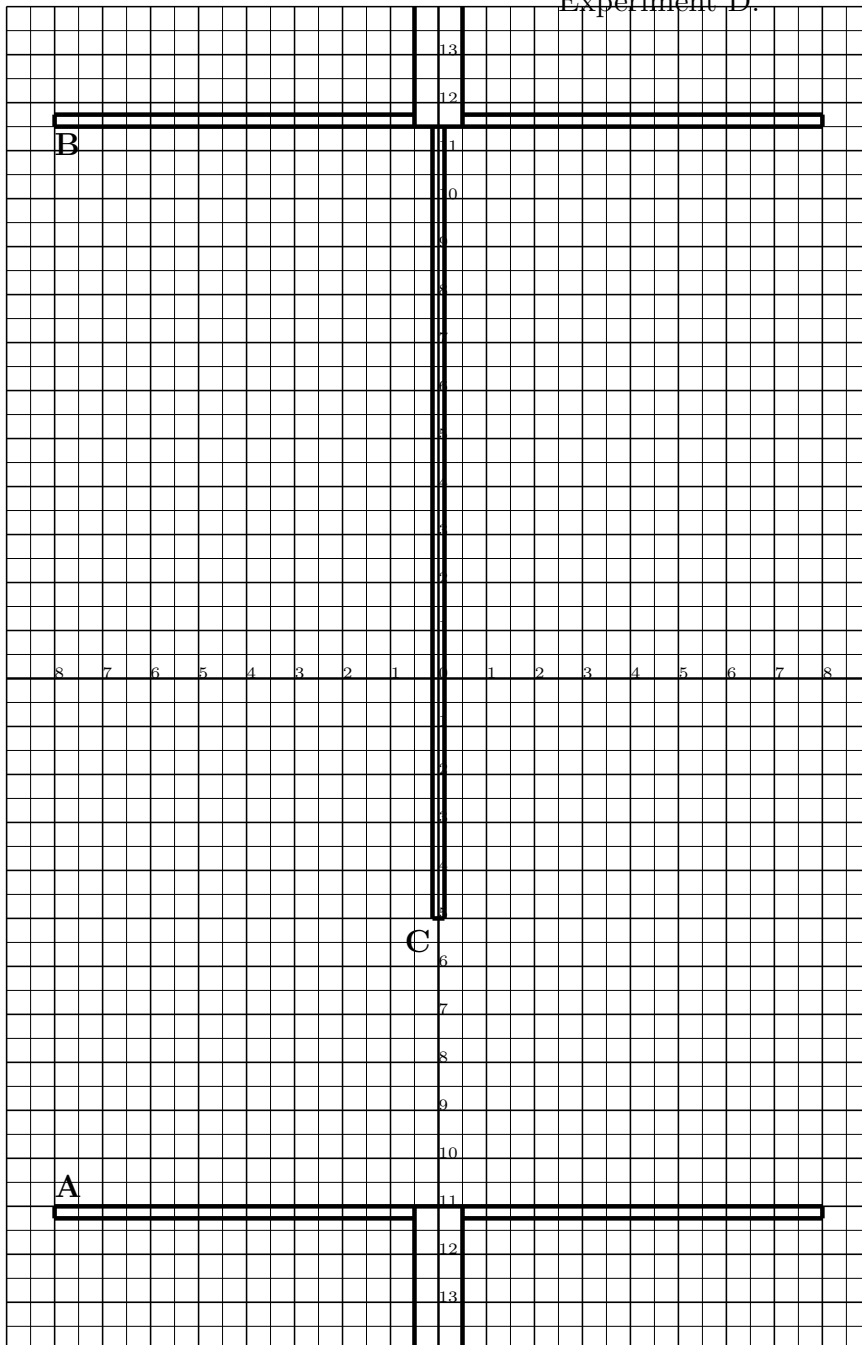


**REPORT SHEET II-5**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_ Partner(s) \_\_\_\_\_

Experiment D.



Shade the region where the electric field is strongest.

blank

- E.** Set up your own configuration of electrodes. You can combine two or more electrodes which previously were used in different experiments. If you want to put two or more different electrodes on the same potential you can connect them with a piece of a wire (keep the wire above the water surface, however). Try to come up with the most intriguing configuration. Before you turn the power supply on make sure there is no direct conductive path from the “+” to the “-” pole. Indicate the position of the electrodes on the Report Sheet II-6. Trace the equipotential lines. Don’t feel constrained to trace 20, 15, 10 and 5 V equipotential lines. You can choose your own values of potential if you wish. At the end, try to draw the lines of force.



**REPORT SHEET II-6**

Date \_\_\_\_\_ Name \_\_\_\_\_

Instructor \_\_\_\_\_ Partner(s) \_\_\_\_\_

Experiment E.

