VI. RC CIRCUITS

f05.01

INTRODUCTION

RC circuits contain capacitors in addition to resistors. Capacitors are devices that can store electric charge. Unlike in DC circuits, time dependence is essential in RC circuits.

PURPOSE

- Experimental observation of time evolution of simple RC circuits.
- Verification of the rules for capacitors arranged in parallel or in series.

PRE-LAB ASSIGNMENTS

A. Readings:

A capacitor is a device that can store electric charge. Usually a capacitor consists of a pair of conductive elements, where charge can accumulate, separated by an insulator (e.g. air gap). The two conductive elements store charge of opposite sign ($-Q$ and $+Q$), thus usually net charge on the capacitor is zero ($-Q + Q = 0$). The accumulated charges create electric field in the insulator. Therefore, there is a drop of electric potential across the capacitor (ΔV). The ratio of charge stored in the capacitor to the potential drop across the capacitor is a *constant* quantity called **capacitance** (C):

$$C = \frac{Q}{\Delta V} \tag{1}$$

Capacitance depends on size and construction details of the capacitor but does not depend on Q or ΔV. A unit of capacitance is *Farad* equivalent to *Coulomb/Volt*: $[F] = [C]/[V]$.

Potential drop across a capacitor, ΔV, must be included when applying Kirchhoff's Loop Rule to the electric circuit containing the capacitor.

Similarly to resistors, two or more capacitors connected in parallel or in series can be logically replaced by one effective capacitor. The rules for effective capacitance of a system of capacitors are *opposite* to the rules for effective resistance. Therefore, effective capacitance of capacitors C_1, C_2, $\ldots C_N$ connected in series can be found from:

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N} \tag{2}$$

whereas, effective capacitance of capacitors connected in parallel is a simple sum:

$$C_{eff} = C_1 + C_2 + \ldots + C_N \tag{3}$$

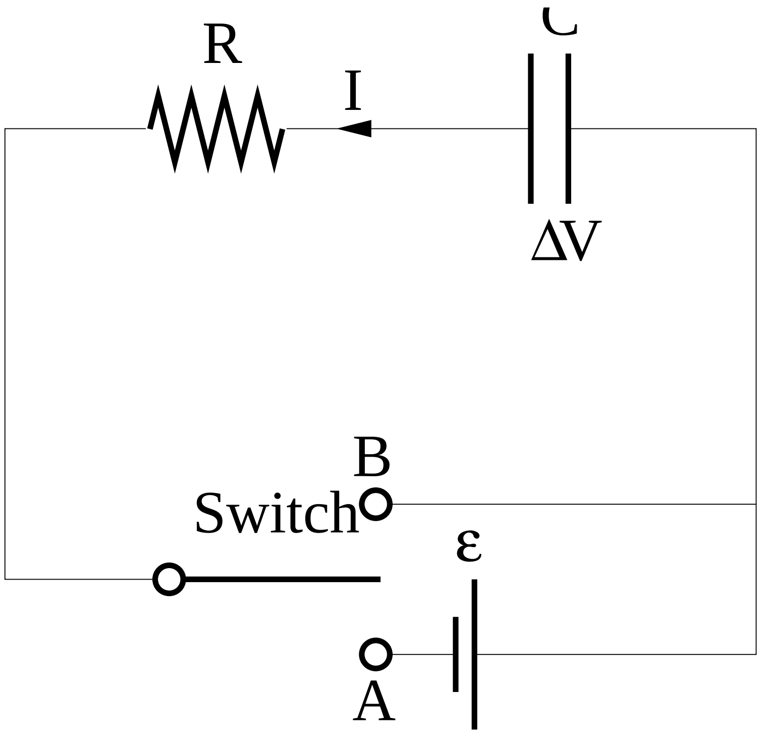

Figure 1. RC circuits for charging (the switch in position A) and for discharging (the switch in position B) capacitor.

A constant electric current cannot flow through a capacitor since the capacitor contains an insulator breaking the conductive path. However, if charge on the capacitor increases or decreases with time, current will flow reflecting rate at which the charge is changing;

$$I = \frac{dQ}{dt} \tag{4}$$

Since the change in Q implies change in ΔV, which in turn effects I via Ohm's law for the circuit connected to the capacitor, the current will change with time too. For the simplest circuit with just a capacitor and a resistor and no other elements, like the circuit shown in Fig. 1 with the switch in position B, the loop rule and Ohm's law say:

$$\Delta V + I\,R = 0 \tag{5}$$

Combing this formula with (4) and (1) we get:

$$\frac{dQ}{dt} = -\frac{Q}{R\,C} \tag{6}$$

thus

$$\frac{dQ}{Q} = -\frac{dt}{R\,C} \tag{7}$$

Now we can integrate by sides and obtain

$$\ln Q = -\frac{t}{R\,C} + \text{const} \tag{8}$$

which can be also written as

$$Q = Q_0 \exp\left(-\frac{t}{\tau}\right) \tag{9}$$

where Q_0 is the charge on the capacitor at $t = 0$, and

$$\tau = RC \tag{10}$$

is the so called time constant of the RC circuit. If R and C are expressed in SI units (i.e. Ω and F) then τ will be expressed in SI unit too (i.e seconds).

In our experiments we will measure time dependence of ΔV and of the current in the RC loop. Formulae for these quantities can be easily obtained from (9), (1) and (4):

$$\Delta V = \Delta V_0 \exp\left(-\frac{t}{\tau}\right) \tag{11}$$

$$I = I_0 \exp\left(-\frac{t}{\tau}\right) \tag{12}$$

Clearly, the simplest RC circuit discharges the capacitor through the resistor in an exponential way. After a long time ($t \to \infty$) there will be no charge nor potential drop left on the capacitor ($Q \to 0$, $\Delta V \to 0$) and the current will stop flowing ($I \to 0$). The time constant τ describes how fast this is going to happen. For example, after time equal to one unit of τ all these quantities will decrease to $e^{-1} = 0.37$ of their initial value. After, $t = 5\tau$ they will be down to $e^{-5} = 0.007$ of the initial values.

If a battery generating potential difference ϵ is a part of the RC circuit, like in the circuit shown in Fig. 1 with the switch in position A, then it will contribute to Kirchhoff's Loop Rule (compare to (5)):

$$\Delta V + I\,R - \epsilon = 0 \tag{13}$$

The current will flow in the circuit until the potential drop across the capacitor cancels the potential difference generated by the battery $\Delta V \to \Delta V_\infty = \epsilon$. Assuming that initially there is no charge on the capacitor ($Q_0 = 0$, $\Delta V_0 = 0$) we have:

$$\Delta V = \Delta V_\infty \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{14}$$

$$I = I_0 \exp\left(-\frac{t}{\tau}\right) \tag{15}$$

The initial current in the loop is easy to calculate from the loop rule (13):

$$I_0 = \frac{\epsilon}{R} \tag{16}$$

It is clearly independent of the capacitor. The rate in which capacitor is charging is again determined by the RC time constant (10). After a long time $\Delta V \to \Delta V_\infty = \epsilon$ and $I \to 0$ as argued above.

B. Exercises:

Please answer the questions on Report Sheet VI–1, which will be collected at the *beginning* of the laboratory session and graded by your instructor.

REPORT SHEET VI–1

Date____________________ Name____________________

Instructor____________________

PRE-LAB EXERCISES

Exercise 1.
If $R = 2M\Omega$ and $C = 3\mu F$ in Fig. 1, what is the value of the time constant for this circuit (specify units in your answer)?

Exercise 2.
Calculate effective capacitance of the two capacitors connected as shown.

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LABORATORY ASSIGNMENTS

In this laboratory we will measure time dependence of the potential drop across a capacitor and of the current flowing through the RC circuit to verify the formulae derived in the theoretical introduction. We will first charge the capacitor from a battery, then we will discharge it by excluding the battery from the circuit. The circuit is shown in Fig. 1. When the switch is in position A the capacitor is charging from the battery and equations (14)-(15) apply. Then we will flip the switch to position B which excludes the battery from the circuit and the capacitor will discharge according to formulae (11)-(12).

The current in the RC loop I will be measured by connecting the voltage probe across 1MΩ resistor, which will also play the role of the resistor R in the circuit depicted in Fig. 1 ($I = V/R$). The voltage drop on the capacitor ΔV will be measured by connecting the second voltage probe across the capacitor.

The voltage probes introduce a slight complication to our circuit since a small current can flow through the voltage probes themselves. Resistance of the voltage probe R_P may differ for each set up and is somewhere between 1.5 and 20MΩ. This is large enough that we could neglect this effect in the previous experiments. However, in this experiment the resistance of the probes has to be taken into account. This modifies our circuit to the one shown in Fig. 2. If you are interested in understanding how the formulae for discharging and charging capacitor change keep reading. Otherwise skip to the experimental procedures which will refer to the formulae to be tested.

Now, with the switch in position B the capacitor discharges not only through the 1MΩ resistor but also through the probes represented by the resistors R_P. Therefore, when calculating expected time constant of the circuit with formula (10) we must use effective resistance R for the 1MΩ resistor and two R_P resistors all connected in parallel (convince yourself that when the battery is excluded, the circuit in Fig. 2 can be redrawn to have all three resistors connected in parallel to the capacitor):

$$R^B_{eff} = \frac{1}{\frac{1}{1M\Omega} + \frac{1}{R_P} + \frac{1}{R_P}} \tag{17}$$

$$\tau \quad [sec] \quad \approx \quad C \quad [\mu F] \quad \cdot \quad R^B_{eff} \quad [M\Omega] \quad . \tag{18}$$

Otherwise the equations (11)-(12) for discharging capacitor are unchanged.

Situation is a bit more complicated with the switch in position A, i.e. when the capacitor is charged from the battery. After a long time ($t \to \infty$) charge on the capacitor will reach its constant value and the current through the capacitor will be zero ($dQ/dt = 0$). However, the current I, which we measure, will not be zero since some current will still flow through the probe connected to the capacitor. Effective resistance connected to the battery is

$$R^A_{eff} = R_P + \frac{1}{\frac{1}{1M\Omega} + \frac{1}{R_P}}, \tag{19}$$

thus this remnant current will be

$$I_\infty \quad [\mu A] \quad = \frac{\epsilon \quad [V]}{R^A_{eff} \quad [M\Omega]} \tag{20}$$

Voltage drop across the capacitor will be the same as voltage drop across the probe:

$$\Delta V_\infty = I_\infty R_P = \epsilon \cdot R_P / R^A_{eff} \tag{21}$$

Assuming that the capacitor is not charged at $t = 0$, the equation (14) is still true, except for the different value of ΔV_∞. However, the current I is now a sum of the current through the capacitor expressed by equation (15) and the current through the probe. Together they give:

$$I = I_0 \exp\left(-\frac{t}{\tau}\right) + I_\infty \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{22}$$

The resistance in the formula for I_0 (16) is due to the 1MΩ resistor and R_P connected in parallel, thus

$$R^{A'}_{eff} = \frac{1}{\frac{1}{1M\Omega} + \frac{1}{R_P}} \tag{23}$$

and

$$I_0 \quad [\mu A] \quad = \quad \frac{\epsilon \quad [V]}{R^{A'}_{eff} \quad [M\Omega]} \tag{24}$$

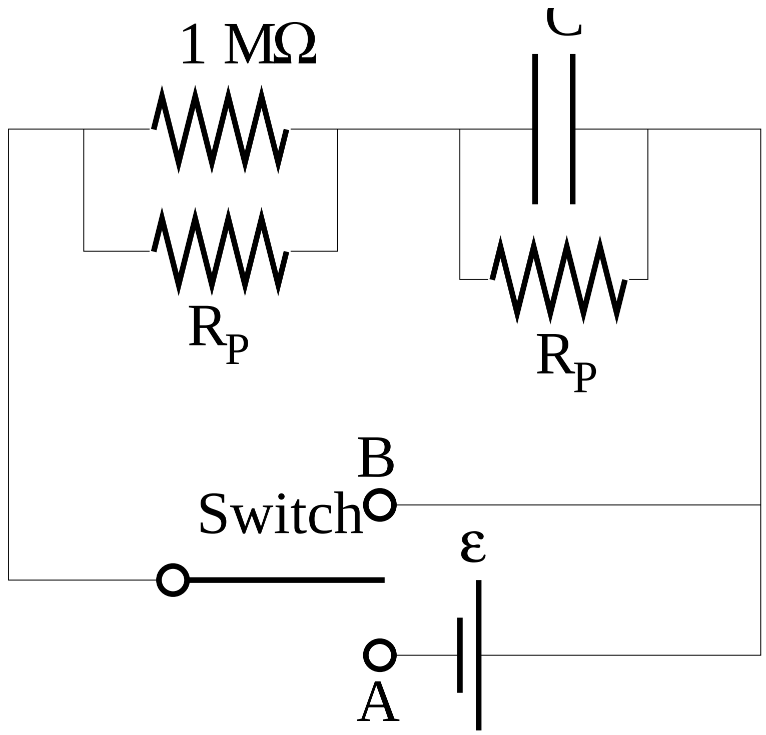


Figure 2. The RC circuit from Fig. 1 with voltage probes connected (represented here by resistors R_P).

Materials Needed:

- capacitors: $0.8\mu F$, $2.2\mu F$, $220\mu F$
- $1M\Omega$ resistor
- $\sim 4.5V$ battery
- Dual Channel Amplifier with two voltage probes
- ULI computer interface box
- Voltmeter (for apparatus test only)
- Cables
- Switch

Experiment A: *Charging of a capacitor*

The Task:

To experimentally observe circuit in which capacitor is charged.

Procedures

A-1. Using handheld meter, measure internal resistance R_P of each voltage probe (the probe should not be connected to anything when you do that). Hopefully, they are not too different. Calculate average R_P value and report it in Report Sheet VI–2. Also check if the probes read voltages correctly when connected to a battery.

A-2. Connect the circuit as shown in Fig. 2. Use $2.2\mu F$ capacitor. The voltage probe 1 should be connected across the capacitor. The voltage probe 2 should be connected across $1M\Omega$ resistor. Make sure the probes are connected to the Dual Channel Amplifier and that the latter is connected to the ULI interface box (DIN1 to DIN1, DIN2 to DIN2). Switch the interface box on. Start the computer, and click on the PHY222 icon to start the program. To load the proper initialization file, choose "Open..." from the "File" menu. Open the file "rc" in PHY222 subdirectory.

A-3. Set the switch to position B and wait a few seconds to discharge the capacitor.

Then start collecting data. After a second or two quickly flip the switch to position A which will start charging the capacitor. Copy graphs ΔV vs. Time and I vs. Time onto Report Sheet VI–2 (don't pay any attention to the other two graphs on the right side on the screen).

The graph ΔV vs. Time illustrates the formula (14). What is the measured value of ΔV after the capacitor saturates? Compare it to the value expected from the formula (21). You will need to calculate R^A_{eff} from the formulae (19) and measure ϵ of your battery with a hand-held voltmeter to perform this comparison. Report your results in Report Sheet VI–2.

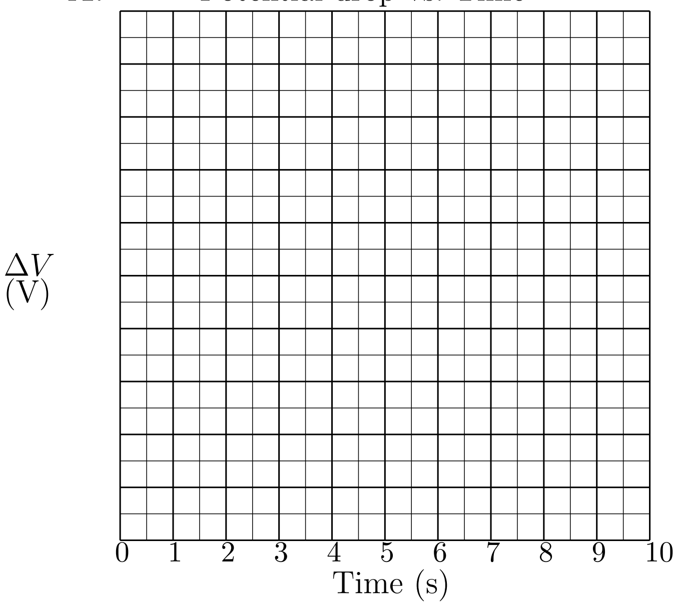
The graph I vs. Time illustrates the formula (22). Read from the graph the initial current value, I_0, right after the switch was flipped to position A. Compare it to the expected value from formula (24). Also read from your graph I_∞ and compare it to the expected value from formula (20). Report your results in Report Sheet VI–2.

REPORT SHEET VI–2

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

A. Potential drop vs. Time



$R_P =$ $M\Omega$

R^A_{eff} from formula (19) = $M\Omega$

ϵ of battery = V

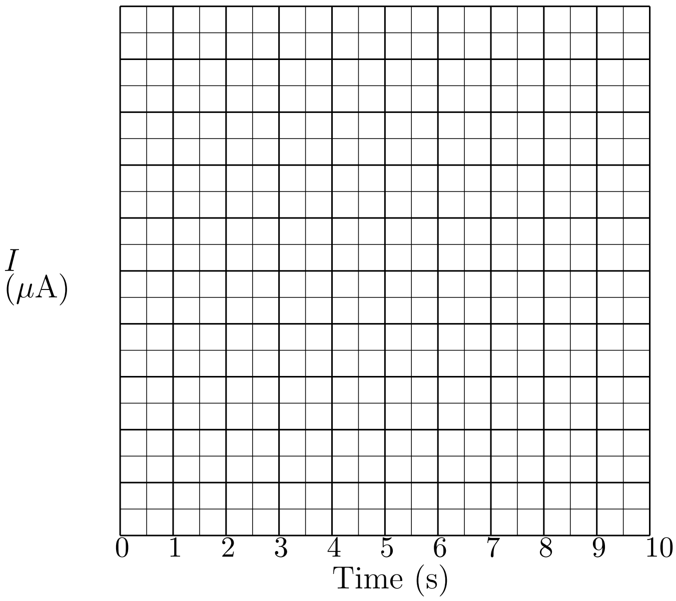
ΔV_∞ from formula (21) = V

ΔV_∞ measured from data = V

Do the measured and expected values of ΔV_∞ roughly agree?

☐ yes ☐ no

Current vs. Time



$R^{A'}_{eff}$ from formula (23) = $M\Omega$

I_0 from formula (24) = μA

I_0 measured from data = μA

Do the measured and expected values of I_0 roughly agree?

☐ yes ☐ no

I_∞ from formula (20) = μA

I_∞ measured from data = μA

Do the measured and expected values of I_∞ roughly agree?

☐ yes ☐ no

blank

Experiment B: *Discharging of a capacitor*

The Task:

To experimentally observe circuit in which capacitor is discharged and to measure its capacitance from the time constant of the RC circuit.

Procedures

B-1. Use the same circuit as in the previous experiment. Set the switch to position A and wait at least 10 seconds to charge the capacitor.

Then start collecting data. After a second or two quickly flip the switch to position B which will start discharging the capacitor. Copy graphs ΔV vs. Time and I vs. Time onto Report Sheet VI–3.

B-2. The other two graphs on the right side on the screen are $\ln(\Delta V)$ vs. Time and $\ln(I)$ vs. Time. Taking logarithm of equations (11)-(12) we expect:

$$\ln(\Delta V) = -\,t\,\frac{1}{\tau} + \ln(\Delta V_0) \tag{25}$$

$$\ln(I) = -\,t\,\frac{1}{\tau} + \ln(I_0) \tag{26}$$

Therefore, both of these graphs should have linear part with slope equal to $1/\tau$. By measuring this slope we can determine τ, which can be then used to determine experimental value of the capacitance.

Click on $\ln(\Delta V)$ vs. Time graph to select it. The part of the graph before the switch was flipped is not linear and must be excluded. If the data becomes erratic for longer times due to the measurement inaccuracies, they must be excluded too. To select a good linear part, left-click on the first point of the graph to be included and with the mouse button still pressed move to the last point of the graph to be included. Release the mouse button. The range to be fit should be indicated by vertical lines. Then go to "Analyze" menu and select "Linear Fit". The box superimposed on the graph shows the fit results (here $y = \ln(\Delta V)$, $x = t$ and m is the slope). Note down the slope value in Report Sheet VI–3.

B-3. Apply the same procedure to measure the slope from $\ln(I)$ vs. Time graph. Note down the slope value in Report Sheet VI–3. Are the two fitted slopes roughly the same as predicted by the theory?

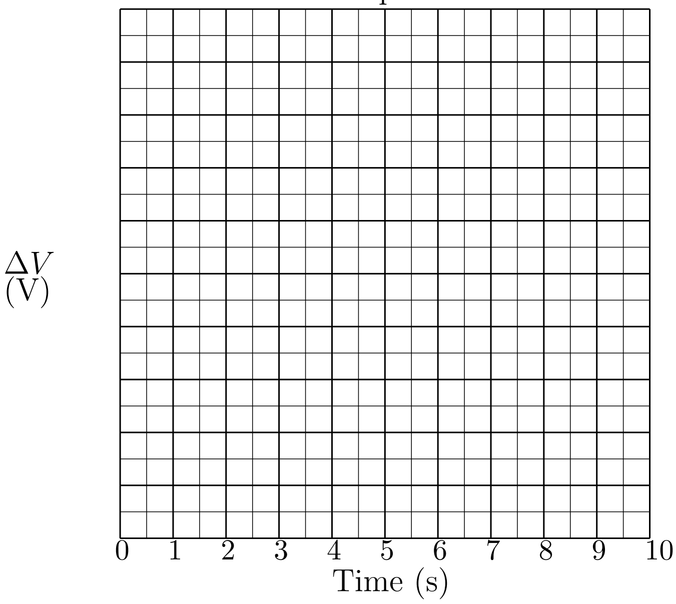
B-4. Calculate the average of the two slopes $m_{ave} = (m_{\ln V} + m_{\ln I})/2$. Then invert this slope into the time constant: $\tau = 1/m_{ave}$. Finally determine capacitance from τ using formula (18). Is it close to the nominal value of $2.2\mu F$? Report all calculations in Report Sheet VI–3.

REPORT SHEET VI–3

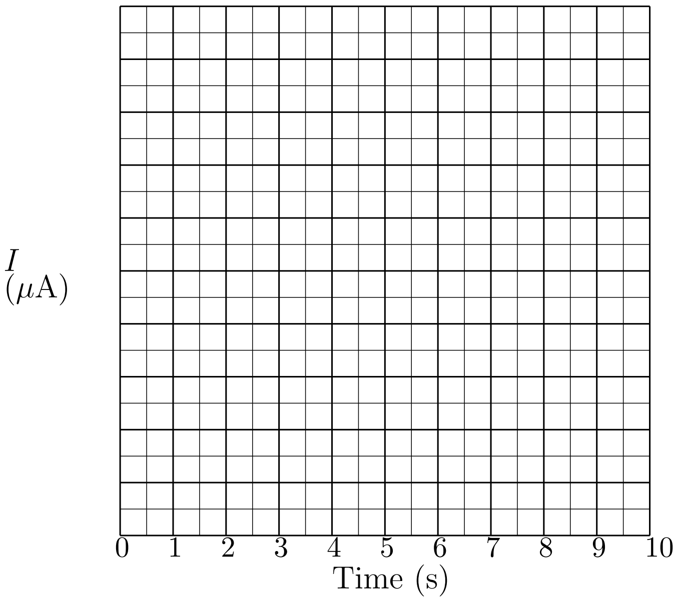
Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

B-1. Potential drop vs. Time



B-1. Current vs. Time



B-2 $m_{\ln V}$ from fit = sec^{-1}

B-3 $m_{\ln I}$ from fit = sec^{-1}

B-4 Average slope m_{ave} = sec^{-1}

$\tau = 1/m_{ave}$ = sec

R^B_{eff} from formula (17) = $M\Omega$

C from τ and formula (18) = μF

Is the measured value of C close to the nominal rating of the capacitor (i.e. 2.2 μF)?

☐ yes ☐ no

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Experiment C: *Faster capacitor*

The Task:

To charge and discharge a capacitor with smaller capacitance.

Procedures

C-1. Replace the $2.2\mu F$ capacitor by the $0.8\mu F$ capacitor in the circuit.

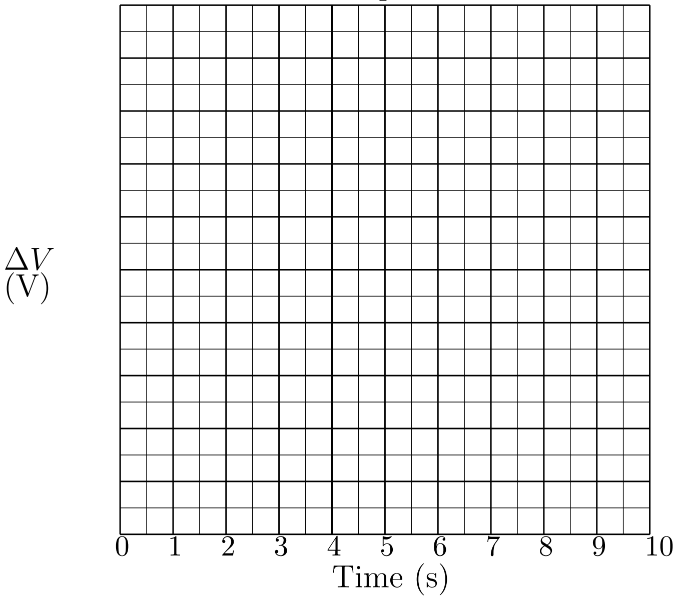
C-2. Follow the procedure outlined in experiment A to charge the capacitor. Report your measurements in Report Sheet VI–4.

REPORT SHEET VI–4

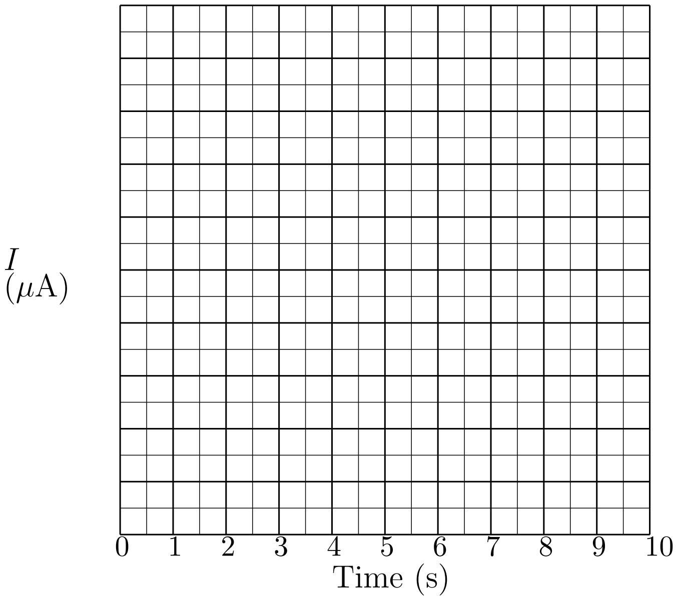
Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

C-2. Potential drop vs. Time



Current vs. Time



ΔV_∞ measured from data = V

According to formula (21) ΔV_∞ should be independent of the value of the capacitance. Is the value measured here in rough agreement with the value measured in Report Sheet VI–2?

☐ yes ☐ no

I_0 measured from data = μA

According to formula (24) I_0 should be independent of the value of the capacitance. Is the value measured here in rough agreement with the value measured in Report Sheet VI–2?

☐ yes ☐ no

I_∞ measured from data = μA

According to formula (20) I_∞ should be independent of the value of the capacitance. Is the value measured here in rough agreement with the value measured in Report Sheet VI–2?

☐ yes ☐ no

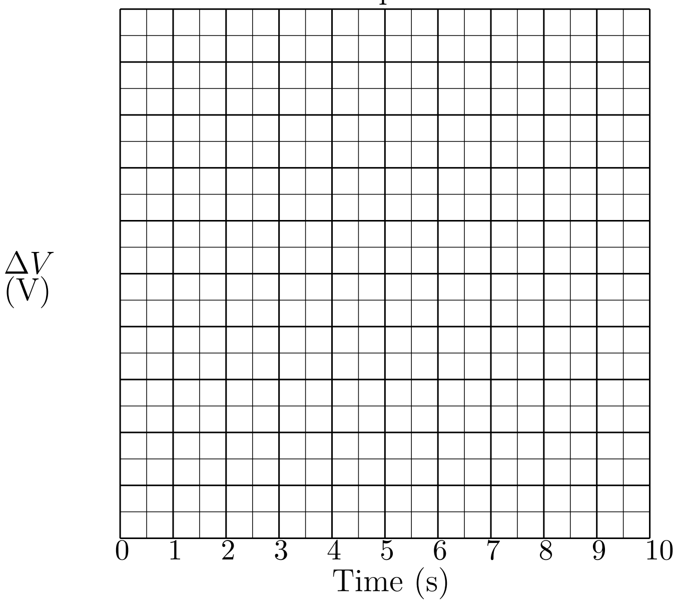
C-3. Follow the procedure outlined in the experiment B to discharge the capacitor. Report your measurements in Report Sheet VI–5.

REPORT SHEET VI–5

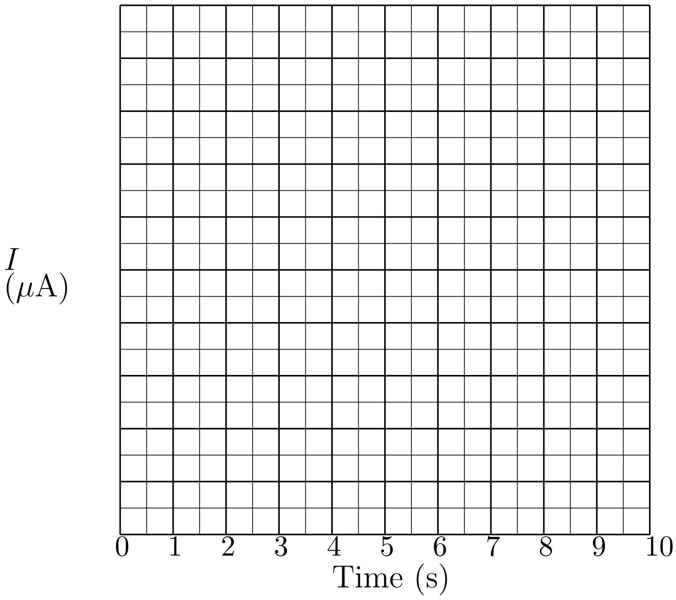
Date________________________ Name________________________

Instructor______________________ Partner(s)______________________

C-3. Potential drop vs. Time



Current vs. Time



$m_{\ln V}$ from fit = sec^{-1}

$m_{\ln I}$ from fit = sec^{-1}

Average slope m_{ave} = sec^{-1}

$\tau = 1/m_{ave}$ = sec

C from τ and formula (18) = μF

Is the measured value of C close to the nominal rating of the capacitor (i.e. 0.8 μF)?

☐ yes ☐ no

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Experiment D: *Slow capacitor*

The Task:

To charge capacitor with much larger capacitance.

Procedures

D-1. Replace the $0.8\mu F$ capacitor by the $220\mu F$ capacitor in the circuit.

D-2. Discharge the capacitor by connecting a wire directly across the capacitor (discharging it through the circuit with the switch in position B would take too long). Then discoonnect this wire and charge the capacitor like in the experiment A. Since the time constant is now very long, in 10 seconds of data taking you observe only the beginning of the charging process. It would take many minutes to fully charge this capacitor. Instead of determining τ and C from discharging the capacitor we will use the graphs for charging the capacitor that you have obtained here. Since in this experiment t is very small compared to τ we can use the following approximation of the exponent in the theoretical formulae:

$$\exp\left(-\frac{t}{\tau}\right) \approx 1 - \frac{t}{\tau} \tag{27}$$

Putting this into formulae (14) and (22) we obtain:

$$\Delta V \approx t\,\frac{\Delta V_\infty}{\tau} \tag{28}$$

$$\Delta I \approx I_0 - t\,\frac{I_0 - I_\infty}{\tau} \tag{29}$$

Therefore, ΔV vs. Time graph should be approximately linear with the slope of

$$m_V = \frac{\Delta V_\infty}{\tau} \tag{30}$$

and I vs. Time graph should be approximately linear with the slope of

$$m_I = -\frac{I_0 - I_\infty}{\tau} \tag{31}$$

Determine both slopes from fits to your data (use linear parts of the graphs only) and report them in Report Sheet VI–6.

Since you do not observe V_∞ and I_∞ in this experiment use theoretical formulae (21) for ΔV_∞, (24) for I_0 and (20) for I_∞ to invert the measured slopes to the time constant using formulae (30) and (31). Are the determined slopes roughly consistent? (see Report Sheet VI–6)

D-3. Average the two measured time constants. Obtain measured value of C from the average τ using the formula (18). (see Report Sheet VI–6)

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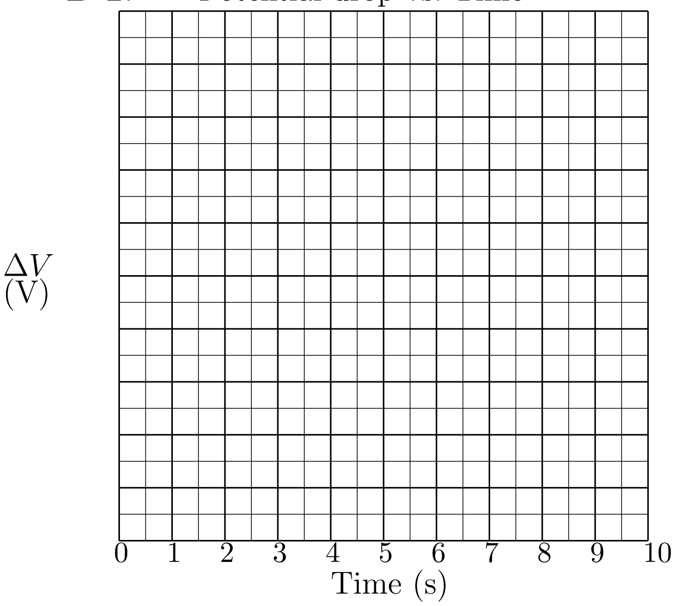
REPORT SHEET VI–6

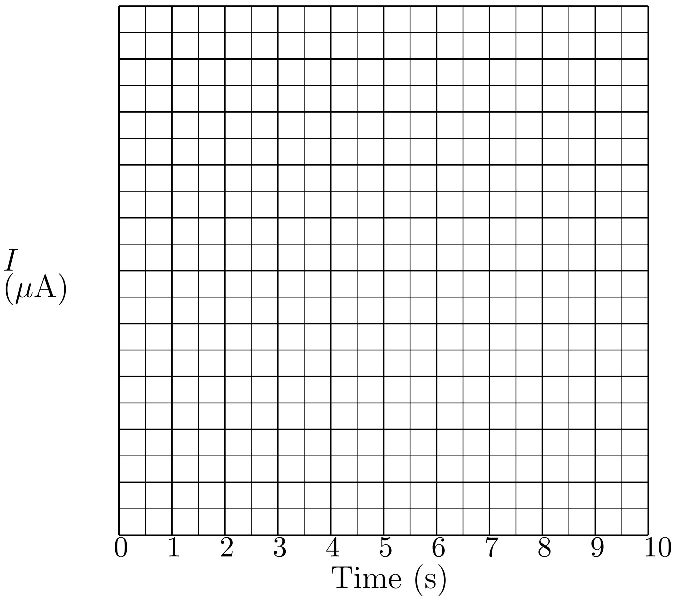
Date__________________________ Name__________________________

Instructor_______________________ Partner(s)_______________________

D-2. Potential drop vs. Time



D-2. Current vs. Time



D-2 m_V from fit = V/sec

ΔV_∞ from formula (21) = V

$\tau_V = \Delta V_\infty / m_V$ = sec

m_I from fit = $\mu A/sec$

I_0 from formula (24) = μA

I_∞ from formula (20) = μA

$I_0 - I_\infty$ = μA

$\tau_I = (I_0 - I_\infty)/m_I$ = sec

D-3 Average time constant τ_{ave} = sec

C from τ_{ave} and formula (18) = μF

Is the measured value of C close to the nominal rating of the capacitor (i.e. 220 μF)?

☐ yes ☐ no

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Experiment E: *Capacitors connected in parallel*

The Task:

To verify the rule for effective capacitance of the capacitors connected in parallel.

Procedures

E-1. Connect the $0.8\mu F$ and $2.2\mu F$ capacitors in parallel to each other replacing single $220\mu F$ from the previous experiment.

E-2. Use the method outlined in experiment B to measure effective capacitance of this system of two capacitors. Compare your result to the expected effective capacitance given by equation (3). Use the measured values of capacitance in experiments B and C rather than the nominal ratings of these capacitors. Report your results in Report Sheet VI–7.

Experiment F: *Capacitors connected in series*

The Task:

To verify the rule for effective capacitance of the capacitors connected in series.

Procedures

F-1. Connect the $0.8\mu F$ and $2.2\mu F$ capacitors in series. The voltage probe 1 should be connected across both of them.

F-2. Use the method outlined in experiment B to measure effective capacitance of this system of two capacitors. Compare your result to the expected effective capacitance given by equation (2). Report your results in Report Sheet VI–7.

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REPORT SHEET VI–7

Date____________________ Name____________________

Instructor____________________ Partner(s)____________________

For convenience copy the measured values of C from:

Report Sheet VI–3 $C_1 =$ μF

Report Sheet VI–5 $C_2 =$ μF

E-2

$m_{\ln V}$ from fit $=$ sec^{-1}

$m_{\ln I}$ from fit $=$ sec^{-1}

Average slope $m_{ave} =$ sec^{-1}

$\tau = 1/m_{ave} =$ sec

C from τ and formula (18) $=$ μF

Effective C from formula (3) (use measured values of C_1 and C_2) $=$ μF

Is the measured value of C close to the expected one from formula (3)?

☐ yes ☐ no

F-2

$m_{\ln V}$ from fit $=$ sec^{-1}

$m_{\ln I}$ from fit $=$ sec^{-1}

Average slope $m_{ave} =$ sec^{-1}

$\tau = 1/m_{ave} =$ sec

C from τ and formula (18) $=$ μF

Effective C from formula (2) (use measured values of C_1 and C_2) $=$ μF

Is the measured value of C close to the expected one from formula (2)?

☐ yes ☐ no