PHY307 HWK ASSIGNMENT #4, due Sept. 17, 2002:

Read *Chaos* through p. 80 (end of Ch. 3, "Life's Up and Downs"). There will not be a reading quiz, but take some notes on what Smale and May each did, for possible testing on an exam. (I'll give out sample questions, later.) 607 students should do problems 1 through 4.

Problems:

1. Chaotic colors. One can also use colors to visualize chaos. Use your map2.py code, and instead of modifying position, modify color. How to do this? The **color** of an object can be set by something like: **objectname.color = (r, g, b)**, where r, g, and b are the red, green and blue components of the object color. So,
   a. Start with map2.py.
   b. Make two cubes, **a** and **b,** side by side, using a box.
   c. Make two variables, **colora** and **colorb.**
   d. Instead of modifying the position of the cubes, modify their color, via, for example,

      a.color = (colora, 0, 1-colora)

      which changes the red vs. blue balance in the color of **cubea.** Similarly, update the color of **cubeb.**
   e. Include your code and note how the changing colors reflect the butterfly effect or not for **a=2.5, 3.5** and **3.8.**

2. Quartic maps. Try a different map. Start with your map2.py code, then replace the formula *x ?   a*x*(1-x)* with *x ?   a*((x-1)**2)*((1+x)**2)*. Keep $a < 1.4$ and start your cones with $-1 < x < 1$. What types of behaviors do you see? Speeding up your loop (faster rate) can be helpful. For what values of *a* do you see these behaviors (explore !)

3. [For PHY607 students] Chaotic Chicle? Assume you have a spherical ball of a quasi-elastic substance and you have a horizontal plate oscillating up and down (y-direction) at frequency ? . There is a gravitational field uniform in the y-direction.You drop the ball onto the surface – when it bounces, it retains a fraction ? of its kinetic energy, in the frame of the moving plate. Here, you will compare the results from integrating the equations of motion with a map derived from the continuous time equations of motion.
   a. Simulate this dynamics in continuous time and search parameter space for simple and chaotic behavior. Use VPython to simulate and visualize the dynamics.
   b. Write down an analytic *map* for this system, which takes the two variables (velocity after the collision and the phase of the plate) to the velocity and phase of the next collision. Plot out the behavior of this map, looking for simple and chaotic behavior for different parameters (?,?).

4. [For PHY607 students] Analyzing sensitivity. For 307, we will not be using much calculus. Here is your chance to do some, coupled with simulations.

   The sensitivity to initial conditions can be precisely defined for infinitesimally separated initial conditions. That is, if one follows two one-dimensional systems differing by ? at discrete time *t*, i.e., *x(t) = z* and *x(t)=z + ?*, after one application of the map, the difference will be *?(t+1) = f'(z)?*. So, by following one point and keeping track of the derivative at each time, one can see how much divergence there will be after *m* applications of the map:

   ?*(t+m) = f'(x(t+m-1)) f'(x(t+m-2)) … f'(x(t))*

   The *rate of divergence* is the Lyapunov exponent. This can be estimated by computing the map and its derivatives for a large number of time steps and then finding

   estimated ? = log(?(*m*)) / *m*

   Estimate ? for the values of *a* used in the lab, by running the map *f(x)=ax(1-x)* for a large number of iterations, given *a*, keeping track of the derivatives (note that for very long runs, it is best to keep track of the sum of the logarithms of the magnitude of *f'*, rather than multiplying the *f'* values together, as this multiplication will diverge, exceeding the floating point representation.) Include your code and conclusions (this can be done in C, if you prefer, rather than Python.) It may be nice to try a number of values, and plot ? vs. *a*.