Homework 4

Due: Thursday 28 September

For this homework you will use your codes `solar.py` and `integrator.py` from lab4 to investigate the motion of a object near a black hole (or any object with a strong gravitational field). In this case the leading correction from General Relativity to Newton's inverse square law is a term like α/r^4.

1. The quantity α must carry units of *length*^{2}. Construct such a quantity using the constants G, M and c^2. This number is called the Schwarzschild radius for the gravitating body. Calculate an approximate value for this quantity for the Sun. Hence estimate the magnitude of the relativistic correction term $\frac{\alpha}{r^2}$ for the Earth's orbit. Clearly to get measurable effects we need very massive and compact bodies – like neutron stars and black holes.

2. In `integrator.py` add python code to implement the force law

$$F = -\frac{Gm_a m_b}{r^2}\left(1 + \frac{\alpha}{r^2}\right)$$

   Include this code in your answer. Set $\alpha = 6.8 \times 10^{21}$ which will allow us to see the effects of this correction term.

3. Now modify the code `solar.py` so as to include only 2 gravitating objects, say the "Sun" and the "Earth". Let the latter's velocity be given by $\mathtt{earth_vel} = \mathtt{1.3} * \mathtt{2} * \mathtt{math.pi} * \mathtt{AU}/(\mathtt{365.25} * \mathtt{24.} * \mathtt{60.} * \mathtt{60.})$

4. You will also want to change the size of the screen to `scene.range=3*AU`

5. Run the simulation and note down the time required for the elliptical orbit to precess through 360 degrees (how many ellipses have been drawn to the screen before the orbital pattern *first* becomes approximately rotationally symmetric - each of these partial ellipses takes approximately 1 year to complete)