Lab 12 - The Ising Model

Thursday 16 November, 2006 - Due: t.b.a.

there was no Lab 11

In this lab, you will study a simple model of ferromagnetism, called the Ising model.

As always, please include in your writeup any sections of Python code you write.

♣Read thoroughly. Every sentence has been very carefully constructed to guide you through this lab.

1 The Plot Thickens...

Start with `lab12_ising-magnetization.py`, which is a slight modification of `lec12_ising.py` shown in the lecture. The lecture code does a Monte Carlo simulation of the two-dimensional Ising Model at temperature $T = 3.0$. The revised code that you will start with already incorporates two changes:

- the simulation is run at various temperatures ranging from $T = 0.5$ to $T = 4.0$
- a plot (using VPython's `gdots`) of the "mean-magnetization per unit volume" is drawn

A description of the changes are commented in the code. (♣ Read the code.)

For each temperature T, the mean-magnetization $< M >= \sum_{s=\{\text{system states}\}} M(s)e^{-E(s)/kT}$ is approximated by

$$< M > \approx \frac{1}{N_C} \sum_C M(C)$$

where C represents the set of MonteCarlo-generated configurations and N_C is the number of configurations. Observe how this is calculated in the code by `magnet`. Here is highlighted structure of the calculation (with some lines omitted):

```
for T in [0.5,1.0,1.5,2.0,2.125,2.25,2.5,2.75,3.0,3.5,4.0]:
    magnet=0.0
    for its in range (0,MAXITS):
        ###GENERATE MONTE-CARLO CONFIGURATION
        ###

        if (its>=WARM):
            dum=0.0
            for i in range (0,L):
                for j in range (0,L):
                    dum += spin[i][j].state
            magnet += abs(dum)

    magnet=magnet/((MAXITS-WARM))
    magnetization.plot(pos=(T,abs(magnet)/(L*L)))
```

(Q1)★ **Based on the above calculation and plotting of the mean-magnetization [as seen in** `lab12_ising-magnetization.py` **], add the necessary lines to calculate and plot the susceptibility** *per unit volume* **on an additional** `gdisplay` **.**

$$<\chi>\approx \frac{1}{L^2}\left(<M^2> - <M>^2\right)$$

(Optional:)★ If you want to check that you are doing this calculation correctly, you might try to hand-calculate and write a short Python program to calculate:

for the set of 10 numbers $k = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$,
the mean

$$\mu = <k> = \frac{1}{N}\sum_{i=1}^{N} k_i$$

and variance

$$\sigma^2 = \left(<k^2> - <k>^2\right),$$

which is formed from the mean-of-the-squares $<k^2>$ and the square-of-the-mean $<k>^2$. Compare the results of your hand-calculation and Python program with these values: $N = 10$, $\mu = 4.5$, and $\sigma^2 = 8.25$.

(Q2)★ **Take a screen capture after the simulation is run for the full range of temperatures.** Note the peak-height χ_{max} in the susceptibility-per-unit-volume vs. temperature graph. (Consult the shell window.)

2 Ising on the plane

(Q3)★ **Using the results from the shell-window, complete the table below for the various values of the linear size** L**.**

L	$\log(L)$	χ_{max}	$\log(\chi_{max})$
8			
10			
12			
14			
16			

(Q4)★ **Plot (using Excel or using VPython [see, e.g., Lab3]) a graph of** $\log(L)$ **vs.** $\log(\chi_{max})$ **and determine the slope of the best-fitting line. Use that slope to determine the critical exponent. Compare with the exact value given in the lecture-notes.**