Lec10

- Self-organized critical phenomena

- Earthquakes, sand piles

Self-similarity and criticality

We have so far seen several examples of systems which exhibit *power law* behavior eg.

- Fractal dimensions. Number of cells needed to cover points of fractal/strange attractor $N(s) \sim s^{D_F}$

- Size of percolating cluster as a function of number of lattice points

- Systems exhibiting *phase transitions*. Later.

In general power laws such as these indicate that the system is a self-similar property. Looks same under *change of scale*.

Mathematics of self-similar systems

Mathematically,

$$N(s) \sim s^{-\alpha}$$

If $s \rightarrow bs$ *form* of this function doesn't change.
Contrast with behavior like $N(s) \sim e^{-s}$
Systems are said to be *critical*.
Do not exhibit a characteristic length scale.
May exhibit *universal features*

Self-organized critical systems

- Usually one needs to *tune* external parameters eg. the percolation probability p to achieve this critical condition. (or the temperature T in a thermal phase transitions)

- Occasionally systems will automatically organize themselves into a critical state without any tuning. Such systems are said to be *self-organized*.

Examples:

- Earthquakes

- Sand dunes

Sandpiles

- Discuss simple model showing self-organization.

- Ignore details of motion/forces on sand grains. Just focus on *essence* of problem.
  - Add sand slowly at one point.
  - Allow system to topple at some point when height of local sand pile gets too big.
  - Transfer excess sand to neighbor points. Reaxamine stability of neighbor points.

Model

- One dimension. Start with flat surface.

- Add single grain at LHS.

  - Check if local slope exceeds some value (1 here). If so topple the sandpile by some amount (say 2 grains) and add to next 2 neighbors.

  - Recheck stability of all points and repeat until no further toppling

- Add more sand and repeat

Observations

After some time distribution becomes *stationary* (does not change with time on the average).
Then ask question: what is the average distribution of avalanches/toppling events in the system after a single grain is added.
See power law!

- What is power ? Is it universal (i.e can I tweak the details of the toppling rules to change it
- Is it the same for a more realistic 2d model, etc

Earthquake model

Earthquakes results from the complex relative motion of separate pieces of the Earth's crust. They appear to happen quasi-randomly and their magnitudes have been observed to satisfy the *Gutenberg-Richter law*

$$N(E) \sim E^{-b} \quad \text{where } b \sim 0.5$$

Here, E is the Earthquake magnitude (roghly the amount of energy released during the quake)

- This power law suggests that they may have self-organizing characteristics

- Indeed we can construct a very simple model similar to the sandpile for discussing them

Model

Consider the surface to be represented by blocks with 2d coordinates (i, j). Each block can move independently of its neigbors with $F(i, j)$ representing the net force on that block.
Start from some random initial state

- Increase F everywhere by a small amount $\Delta F = 0.00001$.

- Check if $F > F_c = 4$ critical threshold for slipping

- If one or more blocks unstable go to

- Let $F(i, j) = F(i, j) - F_c$. Relaxation accompanied by $F(i \pm 1, j \pm 1) = F(i \pm 1, j \pm 1) + 1$

Results

After many iterations system approaches steady state. Earthquakes (measured by number of slipping blocks) of all sizes are seen!

Notice, that again have ignored almost all details of problem.

This is justified after the fact by recognizing that we are searching for self-organized *universal* behavior, which should be independent of such details
But note that this model will *not* give an accurate description of individual earthquakes – merely what happens to very many of them.