Lec11

- Earthquakes – a more realistic model

Recap

- Last week we discussed a very simplified model for understanding the *statistics* of Earthquakes – how many Earthquakes occur of a certain magnitude.

- Saw that the dynamics led to *self-organized* critical behavior – power law distribution of Earthquake sizes (Gutenberg-Richter)

- Today we will try a more realistic simulation which incorporates a model of the frictional and elastic forces acting on the tectonic plates

The model

- One dimensional. Focus on 2 *plates* with some (constant) relative velocity v_0

- Model material between the two plates with series of *blocks* with same mass.

- The blocks are coupled elastically together and to the upper plate (2 different spring constants)

- Friction exists between blocks and bottom plate.

Dynamics

- As the top plate moves it stretches the springs connecting it to the blocks.

- Frictional forces build up to stop the blocks sliding

- At some point one block slips - and will try to pull others along with it – an Earthquake.

- Eventually friction slows the blocks to rest again - the Earthquake is over.

- Measure total energy released. Look at distribution of sizes as before.

Equations of motion

Let x_i be position of block i.

$$\begin{aligned}
\frac{dx_i}{dt} &= v_i \\
\frac{dv_i}{dt} &= \frac{1}{m}\Big(k(x_{i+1} - x_i) + k(x_{i-1} - x_i) \\
&+ F_R + k_p(v_0 t - x_i))
\end{aligned}$$

Looks simple *but* be careful F_R is a very non-linear function.

Friction

Assume simple form: static:

$$F_R \leq -F_0$$

kinetic:

$$F_R = -\frac{F_0}{2}\text{sign}(v_i)$$

Opposes motion