Lec12

- Phase transitions, critical phenomena

- Magnetic systems - Ising model

Commerical break

- Next semester there will be a successor course **PHY300** a.k.a **PHY308**

- Tuesdays/Thursdays 12:30-1:50 pm (lab times to be decided)

- Similar to **PHY307** with additional topics drawn from
  - Monte Carlo methods in statistical physics
  - Computational methods in quantum mechanics
  - Fields and waves

Phase transitions

- Many systems composed of (very) many degrees of freedom exhibit *phase transitions*

- These are abrupt changes in the macroscopic state (appearance, properties etc) of the system as some parameter is changed.

- Historically that parameter was often the temperature eg

  - Solid-liquid transition at some critical T_c

  - Transition from magnetic to non-magnetic material for some T_c

  - Cluster percolation at some $p = p_c$

Critical Phenomena

- Close to the phase transition ($T \sim T_c$) the system exhibits power law behavior (compare: self-organized critical systems which require no tuning of parameters).
  - Spanning cluster exhibits structure at all length scales
  - Power law distribution of fluctuations of magnetisation in magnetic material
- More generally a critical system possesses no intrinsic length scale and exhibits *universal* features in various quantities – eg power laws where the numerical value of the power is the same for many systems with differing microscopic dynamics.
- This universal behavior is termed *critical behavior*

Magnetic systems

- Many ferromagnetic materials may possess permanent magnetization

- Every atom contains circulating electrons. These yield small magnetic fields. Sometimes these can add to give a large macroscopic magnetic field – it is said to be a permanent magnet.

- However if the temperature is raised this will in general disappear – the system goes from ferromagnetic to paramagnetic.

- This is a phase transition – close to the transition may different magnetic materials exhibit universal behavior.

Magnetic systems II

- Various thermodynamic quantities diverge or have singular power law behavior there

- This is driven by the system exhibiting *correlations* between widely spaced elementary magnetic domains.

Critical exponents

- Specific Heat $C = \frac{\partial U}{\partial T}$. Near phase transition $C \sim (T - T_c)^{-\alpha}$

- Magnetic susceptibilty $\chi = \frac{\partial M}{\partial T}$. Near phase transition $\chi \sim (T - T_C)^{-\gamma}$

- Magnetization $M \sim (T - T_c)^{\beta}$

Model

- Simple model for these magnetic systems is the *Ising model*.

- Place elementary magnets on sites of simple lattice (representing crystalline structure of material).

- Allow these elementary magnets s_i to point in just 2 possible directions – up and down $s = \pm 1$.

- Allow the energy for the system to be given by

$$E = -J \sum_{<ij>} s_i s_j$$

Dynamics

- Can write/solve dynamical equations – but *very* many atoms in material – too cumbersome and not necessary

- Suffices to have a theory which describes only the *probability* of finding the system in some state – *statistical mechanics*

- Take as basic assumption of this theory that:

  Probability of finding the system in some state with energy E at temperature T is given by $e^{-\frac{E}{kT}}$

- Observables computed by averaging over all possible states using this probability

Examples

- Mean magnetization M

$$< M >= \sum_{\text{states}} M(s) e^{-E(s)/kT}$$

- State of system corresponds specifying the state of each elementary magnet or *spin* on some lattice.

- *Impossible* to do this sum exactly even with a computer.

- Resort to *Monte Carlo* methods

Monte Carlo

- Use a simple algorithm to move from state i to state j.

- Design that algorithm to ensure that after some iterations the probability of any state occuring is just $e^{-E/kT}$

- Measure observables by simple averaging over this set of states. Yields eg. $< M >= \frac{1}{N} \sum_{\text{configC}} M(C)$ with statistical error that varies as $1/\sqrt{N}$ for N states

Metropolis algorithm

Simplest algorithm/update procedure for Monte Carlo

- Pick a site. Try to flip the spin $s \to -s$ Compute change in energy under such a flip ΔE. Local.

- Accept the move with probabiliy $e^{-\frac{\Delta E}{kT}}$

- Keep going ...

Phase transitions in Ising model

- Simplest case - two dimensions.

- Find for $T = T_c = 2.269$ fluctuations in M have a peak.

- $M \sim 0$ for $T > T_c$. $M = 0$ for $T < T_c$

- Close to T_c, $\chi \sim (T - T_C)^1.875$ in 2 dimensions. $M \sim (T - T_C)^{0.5}$