

Lec4

- Many particles - Newtonian gravity
- Solar system simulations

Inverse law

- Newton's *Law of Universal Gravitation*:
- Between two bodies with (gravitational) masses M_a and M_b distance r apart there is a force of *attraction* acting along their relative displacement vector of magnitude GM_aM_b/r^2

$$\mathbf{F}_{ab} = -\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}$$

- Comments: (instantaneous) action at a distance, many bodies add up all (vector) forces to get resultant gravitational force.

New integrator.py

```
def force(..)
def totalacc(..)
## verlet update
def update(objlist, dt):
    for i in objlist:
        i.acc=totalacc(i,objlist)
    for i in objlist:
        i.pos=i.pos+dt*i.velocity+i.acc*dt*dt*0.5
        i.track.append(pos=i.pos)
    for i in objlist:
        i.velocity=i.velocity+
        (i.acc+totalacc(i,objlist))*dt*0.5
```

solar.py

```
from visual import *
from integrator import *
scene.autoscale=0
scene.range=1
Earth=sphere(..)
Sun=sphere(..)
Moon=sphere(..)

# create list of gravitating objects
bodies=[Earth,Sun]

dt=30*60.0*100.0
while True:
    rate(100)
    update(bodies,dt)
```

Comments

- Verlet alg used in integator
- Use real parameters - choose dt small compared to characteristic orbital time scales.
- As in lab3 need to exaggerate radius of Earth, Sun etc to see them with finite screen resolution.
- To get stable circular orbit need to choose initial conditions carefully.

Simple orbits

Stable orbit requires gravitational force to supply *radial* acceleration

$$GM_S M_E / R^2 = M_E v^2 / R = M_E \left(\frac{2\pi}{T} \right)^2 R$$

v_{initial} is tied to r_{initial}

Change v_{initial} (factor of 1.3) – ellipse or ...
escape

Find Kepler's law $R^3 \sim T^2$ – lab4

Why inverse square ?

Try changing force law to $1/r^\beta$

Now orbit does not close even for $\beta = 2.02 \dots!$

Precession

Field line picture leads to inverse square law

Jupiter

Add Jupiter – 1/10 mass of Sun at 5.2 AU.

Stable approx circular orbits still obtained.

But – if crank up mass of Jupiter see orbits don't quite close ... the orbit rotates in time *precession* again

Simple consequence of 3-body system – Jupiter disrupts simple orbit.

If Jupiter were bigger could disrupt orbit completely Such effects are known in Solar system (asteroid belt)

Einstein's GR

Similar effects result from the leading order correction from Einstein's theory of General Relativity

$$F = \frac{GM_S M_E}{r^2} \left(1 + \frac{\alpha}{r^2}\right)$$

GR: $\alpha = 1 \times 10^{-4}$. Here, blow up $\alpha \sim 0.1$ to see ...

Also, use units in which $GM_S = 4\pi^2$ and measure distance in AU and years - ensures $1/r^2$ not zero compared to unity on computer!

Mercury's precession

For Mercury precession of perihelion was a problem for Newtonian gravity – calculated effects of Jupiter and inner planets insufficient to explain magnitude

523/3600 degrees per century as opposed to 566/3600 observed.

General Relativity supplied the difference!