Lec9 – mini lecture

- More fractals – Julia sets

Julia sets

Consider the very simple non-linear map

$$x_{n+1} = x_n^2 - \frac{3}{4}$$

For most starting values of x the final $x = \infty$!
In fact x only remains *bounded* if $|x| < 3/2$
See by drawing graphs of $y = x$ and $y = x^2 - \frac{3}{4}$

- The boundary between the two regions in x (diverging and non-diverging) is called the **Julia set** of the map and contains seemingly 2 points $x = -\frac{3}{2}$ and $x = \frac{3}{2}$

- Things *much* more interesting if we allow ourselves to consider *complex numbers*

Complex numbers

Summary:

- Complex number has 2 parts – real and imaginary

$$z = x + iy$$

- Needed to give answer to question: what is square root of a negative number.

- Add/subtract by adding/subtracting corresponding parts

- Multiply out using usual rules and collect terms *together with* the simple rule $i^2 = -1$

- Magnitude $|z| = \sqrt{(x^2 + y^2)}$

Maps of complex numbers

Consider previous map for complex numbers

$$z_{n+1} = z_n * z_n - \frac{3}{4}$$

- What is now the region in which $|z|$ diverges under iteration of the map ?

- The region of convergence is called the *filled-in Julia set B* and the boundary between between diverging and non-diverging sets is the true *Julia* set of the map.

- Remarkably it is a fractal !!

- The boundary is *rough* on all scales.

Other Julia sets

Try general maps $f(z) = z^2 + a$ with

- $a = -0.85 + 0.18i$
- $a = -1.24 + 0.15i$
- $a = -0.16 + 0.74i$

In lab will add zooming feature which will demonstrate this structure on all length scales.