

List of project topics:

1. Precession of Mercury's perihelion due to General Relativity. An extension of an earlier homework problem. To the usual inverse square gravitational force add a term  $\frac{\alpha}{r^4}$ . Compute the rate of rotation of the resulting elliptical orbit as a function of alpha and plot your results. Finally extrapolate to the physical value of alpha (what is this?).
2. The logistic map in a chaotic regime. Extension of an earlier lab. Measure quantitatively the rate of divergence of two trajectories differing only slightly in their initial conditions in a chaotic regime. This divergence is approximately exponential in time  $\Delta x \sim e^{Lt}$  where  $L$  is termed the Lyapunov exponent. Measure the latter for the logistic map for one or more values of  $r$ . Measure also the Feigenbaum delta corresponding to the onset to chaos in the logistic map improving on your earlier lab result by including 16 and 32 period motions.
3. Consider a 3d box containing 2 moving balls. Use as a basis your lab code or the python code `bounce2.py` included with the VPython distribution. Both of these codes assume the balls make elastic collisions with the walls but experience no mutual forces. Building on your experience with coding the Solar System simulation experiment with adding a repulsive force term acting between the balls. Try the simple form  $F = F_0 \exp(-(r/a)^2)$  where  $a$  is the radius of the balls. Tune the magnitude of the initial velocity with the time step and  $F_0$  to ensure the balls never pass through each other. Show qualitatively that the motion is chaotic.
4. Discuss the simple model of a self-organizing sandpile introduced in class and lab. Investigate how the exponent governing the distribution of avalanches depends on system size. Generalize your model and code to two dimensions. Start from the 1D code used in lab.
5. Experiment with the quasi realistic one dimensional earthquake simulation code we introduced in lab. Investigate how the distribution of earthquake sizes depends on the lattice size used. Compare your results with the simple model discussed in lecture (the python code for that 2D simulation is posted amongst lecture 10 on the website. Can either of these codes produce a power dependence compatible with the Gutenberg-Richter law describing the distribution of earthquake magnitudes on the Earth (see lecture).
6. Investigate the 2d Ising model using the simple Metropolis algorithm. Your answer should include measurements of the magnetization, internal energy and specific heat as a function of the inverse temperature. Discuss what is meant by the phrase "the system undergoes a phase transition".