Jan. 2003

Chaos in a Compass Needle   Phy 344

Introduction:

One might think that ordinary mechanical systems such as a pendulum or swing would have become completely understood in the hundreds of years since Isaac Newton first developed methods for writing down equations describing their motion. Indeed many features of these systems are very well understood, as you may recall from your studies of harmonic oscillators, pendulums, and planetary orbits.

However, in the last twenty years or so it has become clear that the solutions to the equations of classical physics are not well understood if they are "nonlinear." A *linear* equation has the following property. Let us say that some solution to the equation is known: perhaps you have solved the equations to obtain some displacement $x(t)$ as a function of time. For a linear equation any multiple $ax(t)$ is also a good solution to the equation. Some important details about "homogeneous" equations and solutions have been skipped here.

When the equations are "nonlinear," they can describe motions of incredible richness; such motions are often described as *chaos*. Furthermore, even with excellent precision in describing the system at some initial time, the motions at subsequent times become almost unpredictable; the tiniest errors in the initial description become magnified as the system evolves.

In this experiment you will study the motion of a compass needle (a weak magnetic dipole) in a magnetic field which oscillates at one cycle per second. You will first study the "free" oscillations of the compass needle in a steady magnetic field to verify that you understand the equations describing its motion. You will then study the effects of a field oscillating sinusoidally in time; you can adjust the strength of the oscillating field. At each field-strength you will study the compass needle's motion; as you increase the field, you should ultimately see chaotic motion. If time permits you may be able to study the motions more quantitatively by using a stroboscope, which enables you to observe the angular position of the compass needle at a definite point in the cycle of the oscillator.

Suggested Reading:

1. This experiment is described by H. Meissner and G. Schmidt *Am. J. Phys.* **54**, 800 (1986). The article assumes familiarity with nonlinear physics; all you will need is the experimental description.

2. The equation describing the compass needle's motion is based on the torque

experienced by a magnetic dipole in a magnetic field. Please find the discussion of this subject in your elementary physics textbook and review it. You will also need the theory of the simple pendulum to derive one of the requested results.

Suggested Apparatus:

1. Any air-filled compass.
2. Small Helmholtz coils and compass mount. Teltron Model 501 Stand and Model 502 Helmholtz coils. Distributed by Tel-Atomic, Inc..
3. Wavetek AC signal generator.
4. DC power supply. No manual available.
5. Small permanent magnet.
6. Oscilloscope. Hitachi.
7. Digital multimeter
8. Stopwatch. The stopwatches incorporated into digital watches are fine; consult the staff if you don't wear such a watch.

Safety Considerations: The stroboscope uses a high voltage; do not use it without consulting the staff.

Avoiding Damage: The compass needle will be demagnetized or damaged if magnetic fields greater than a few Gauss are employed. Please keep the voltage applied to the small Helmholtz coils below 0.5 Volts (positive or negative). When using the small permanent magnet to "start" the compass needle's motions be careful not to bring the permanent magnet closer than necessary to the needle.

Instructions - Theory of Compass Needles

- Meissner and Schmidt's article presents the following equation to describes a compass needle's motions in an oscillating magnetic field:

$$I\frac{d^2}{dt^2}\theta(t) = m(B\sin(\omega t))\sin(\theta(t). \tag{1}$$

  I is the moment of inertia of the compass-needle about its suspension point; m is the magnetic dipole moment of the needle; $\theta(t)$ is the angular displacement of the needle away from the axis of the magnetic field, and $B\sin(\omega t)$ is the magnetic field strength as a function of time. Derive this equation in your notebook. Be prepared to discuss why this equation is "nonlinear."

- The free oscillation frequency f_0 of the compass needle in a steady magnetic field is given by the expression:

$$f_0^2 = \frac{mB_{dc}}{4\pi^2 I}, \tag{2}$$

where m is the magnetic dipole moment of the compass needle, I is its moment of inertia, and B_{dc} is the magnetic field. Derive this result from equation (1). [*Hint:* Use the same approach as is normally used for the angular displacement of a simple pendulum; you will need the small angle approximation $\sin(\theta) \sim \theta \quad (\theta << 1)$].

Instructions - Free Oscillation Study

1. Align the Helmholtz coils so that their field is parallel to the Earth's magnetic field (ie. parallel to the compass needle's direction). Check that the Helmholtz coils are correctly wired so that the magnetic fields from each of the two coils add (instead of canceling each other). Check the wiring of the signal generator to the Helmholtz coils; make sure that the oscilloscope and voltmeter are correctly wired to measure the voltage output of the generator. Have your wiring checked by a staff member before turning the signal generator on.

2. Measure the dependence of the free oscillation frequency f_0 upon a DC voltage V_{dc} applied to the Helmholtz coils. You can use the small permanent magnet to start the compass needle's free oscillation for each value of voltage; time a few oscillations with a stopwatch to determine the period of oscillation.

3. The magnetic field B generated by the Helmholtz coils should be proportional to the voltage V; assume that the constant of proportionality is β (ie. $B = \beta V$). The equation above for the free oscillation frequency can be written:

$$f_0^2 = \frac{m\beta}{4\pi^2 I} V_{dc}. \tag{3}$$

Graph your data on f_0 vs. V_{dc} to test this expression; choose axes so that the expected graph is a straight line. If the theory is correct, use your graph to determine (i) the value $m\beta/4\pi^2 I$ and (ii) the value of the voltage across the coils for which the Earth's magnetic field and the coils' magnetic field cancel.

Instructions - Visual Study of Forced Oscillations

1. Use the offset function of the signal generator to cancel the Earth's magnetic field as much as possible; you should be able to do this using the data acquired from the last experiment.

2. Set the generator frequency to 1 Hz (cycle per second). Classify the motions of the compass needle as the ac amplitude of the generator is varied using a fixed frequency. For example, at very low voltages the needle will just wiggle a little; at somewhat larger voltages the needle will circle around completely. You will need to "start" various motions by using the small permanent magnet; the procedure is similar to that you used for the free oscillation measurements.

3. The ac voltage amplitude in the last section V_{ac} can be converted to the variable A described by Meissner and Schmidt using the equation:

$$A = \frac{m\beta}{4\pi^2 f^2 I} V_{ac}. \tag{4}$$

   where f is the generator frequency. Using the estimate of $m\beta/4\pi^2 I$ from the last experiment, convert the values of V_{ac} from the last section's measurements into values of A.

4. 4. On a piece of graph paper, lay out a horizontal axis to accomodate the range of A values you studied. Indicate on this axis the values of A for which you noticed changes in the types of motion of the compass needle (ie. "period doubling," "chaos," etc.). Compare your transition values with those reported by Meissner and Schmidt for the "undamped" compass needle.

Additional Experiments

If time permits, try one or more of the following experiments.

1. An important parameter of the compass needle which was not measured above is its "damping" constant, which has an important effect on the "route to chaos" of the compass needle. Meissner and Schmidt suggest one procedure for measuring the damping constant based on the smallest ac voltage V_{ac} required to make the compass needle circle continously. Other possibilities involve measuring the decay of the free oscillations of the compass needle or to use forced oscillations of the needle when the Helmholtz coils' field is perpendicular to the Earth's. Measure this constant and compare with Meissner and Schmidt.

2. You can also measure transition points for other frequencies; the theory you are using predicts that the dividing points on the A-axis you gave in the last section should be the same for other frequencies. Try using a frequency of 1.25 Hz or 0.7 Hz and see if the theory describes your results.

3. Design a procedure for using the large Helmholtz coils to cancel the vertical component of the Earth's magnetic field. Repeat enough of your measurements of forced oscillation to decide if imperfect cancellation of the Earth's field affected your results.

4. Make a stroboscopic study of the compass needle's motions as a function of the amplitude of the magnetic field. The strobe will flash once or twice for each cycle of the generator. You should be able to read the compass needle's angular position or positions at the instants in which the the strobe flashes. Make a graph of these angles as a function of ac amplitude.