Feb. 2001

Magnetic Effects in Matter — Phy 344

Introduction:

This experiment is primarily intended to familiarize you with three important topics in magnetism:

- Electromagnets based on iron.
- Magnetic fluxmeters based on Faraday's law.
- The Hall effect in semiconductors.

Each of these three topics will require that you understand a different type of physics. For electromagnets you generalize the basic idea that currents generate magnet fields to include the effects of strongly magnetic materials - in this case "soft" (ie. magnetically soft) iron. The fluxmeter is a device which measures magnetic flux by exploiting Faraday's law: a coil which cuts magnetic field lines will generate an electromotive force. Finally the Hall effect pertains to to the motions of the mobile charges inside matter when they experience both electric and magnetic forces.

Suggested Reading:

1. Jerry B. Marion and William F. Hornyak, *Principles of Physics* (Saunders, New York, 1984), sections 31-4, 31-5, and Enrichment Topic B. This textbook offers a brief treatment of magnetic materials and electromagnets. A copy is filed in the lab.
2. Appendix A: "Magnetic Fluxmeter," unpublished notes. Your elementary physics textbook probably discusses related topics in the chapters on Faraday's law.
3. Appendix B: "The Hall Effect," unpublished notes. Your elementary physics textbook probably contains a similar description of the Hall effect.

Suggested Apparatus:

1. Electromagnet and power supply (Spectromagnetics, Inc. model 1019 4 inch adjustable gap electromagnet; Spectromagnetics, Inc. model 6001 power supply; manuals on file).
2. Magnetic Fluxmeter (fabricated at SU; see file at SUPHYS).

3. Oscilloscope.
4. Capacitor (1 μF), resistor (10^6 Ω), adapters for construction of analog integrator.
5. Indium arsenide crystal (Ohio Semitronics, Inc. model HR72 Hall effect probe; data sheets on file). The crystal has been mounted on a piece of circuit board and cabled onto a block containing four fuses and banana jack connectors.
6. Power supply (for use with Hall probe).
7. Rheostat. Mulvey 398 Ω; no file.
8. Multimeter

Safety Considerations: No known hazards; the electromagnet will ruin a mechanical watch and will attract iron screws, etc.. Most scientists do not worry about harm caused to the body by these magnetic fields, although you may wish to avoid putting your hand in the electromagnet when it is on.

Avoiding Damage:

- Please do not use more than 100 mA through the Ohio Semitronics Hall effect sensor.
- Do not operate the electromagnet above 30 Amps.

Instructions - Fluxmeter Physics

1. Study the attached description of the magnetic fluxmeter in Appendix A. Find an expression relating the magnetic field B through the coil to the initial voltage ΔV measured on the oscilloscope in terms of the number of turns N in the coil, the area A of the coil, and the circuit parameters R_L, R_o, and C. What is the effect of the angle θ between the magnetic field $\vec{B}$ and the plane of the fluxmeter coil?
2. Build the circuit shown in the fluxmeter notes using $C = 1$ μF and $R_L = 10^6\Omega$. Look up the value of R_o.

3. Turn on the electromagnet using the maximum allowed current value (30 Amps). Set up the oscilloscope to measure the waveform on the RC integrator as you push the coil between the pole pieces of the magnet and then pull the coil out. Record these waveforms roughly in your notebook. Make sure you understand how to obtain ΔV from the oscilloscope, and adjust the oscilloscope's settings to make this as easy as possible. In particular you should consider adjusting the *triggering* of the oscilloscope.

4. Does the waveform you measured on the oscilloscope agree adequately with the theory for the fluxmeter described in Appendix A?

Instructions - Field of an Electromagnet

1. Adjust the gap between the pole pieces of the electromagnet to 0.25 inches (just wide enough to accommodate the fluxmeter). Use the fluxmeter to measure the magnetic field generated by the electromagnet as a function of the current (up to 20 A) through its windings. When you finish measuring go back and remeasure the first point you took to find out how reproducible it was. Then remeasure the last point.

2. Repeat your measurements of the field vs. current of the electromagnet for gaps between the pole pieces of 0.5, 0.75, and 1.0 inches; use the same current values as for step 1. above. Graph all of your data these results on a single plot, including error bars as appropriate.

3. Make a graph showing the magnetic field as a function of the gap between the pole pieces for several winding currents.

4. Can you account for your data on the magnetic field between the pole pieces as a function of winding current and of the gap between the pole pieces with the theory described in Marion and Hornyak? Is the magnetic field proportional to the current through the windings?

Instructions - Hall Effect

1. Make a drawing of the Hall probe in your notebook. Attach the probe to one of the pole pieces using tape. Design a circuit for performing Hall effect measurements; you will need to know the resistance across the probe. Wire the circuit and have it checked by a staff member before starting work on the Hall effect.

2. For a specific current through the Hall probe, measure the dependence of the Hall voltage upon the size of the magnetic field (as measured in the previous section of this experiment). Graph these data, and use the information to estimate the drift velocity of the carriers and the sign of the carriers' charge (positive or negative).

3. For a specific magnetic field, measure the dependence of the Hall voltage upon the current through the probe. For later use you should also record the voltage across the specimen required to drive each current through the probe. Graph the dependence of the Hall voltage upon current, and prepare a second graph to illustrate the relationship of the current density j (in Amperes per square meter) and the drift velocity. Use this last graph to estimate the density of carriers n in the material.

4. Using the data acquired in the last step, graph the dependence of the drift velocity upon the *bias* electric field across the specimen. If possible, use these data to estimate the carrier mobility in the probe. Compare your result to a literature estimate for InAs.

Appendix A Magnetic Fluxmeter Notes by E. A. Schiff, January 1992.

A reasonably simple way to estimate the field of a magnetic is to quickly pull a coil of wire out the magnet. According to Faraday's law from electromagnetism, the change in magnetic flux through the coil will cause an electromotive force $\mathcal{E}$ across the coil which can be measured directly using an oscilloscope. You may wish to try this experiment; however, to actually measure the change in magnetic flux we need to integrate $\mathcal{E}$ over time:

$$\mathcal{E}(t) = -\frac{d\Phi(t)}{dt}$$

$$\int_0^t dt'\mathcal{E}(t') = -(\Phi(t) - \Phi(0)) \equiv -\Delta\Phi$$

The simplest way to evaluate this integral is to integrate the current $i(t)$ which flows through the coil as it is pulled out of the field. We assume that a resistor R_L is wired in series with the coil, so $i(t)R_L = \mathcal{E}(t)$; we are assuming that this current doesn't create any significant magnetic flux through the coil compared to the external magnetic field. Since the integral of a current $i(t)$ is just the electrical charge ΔQ which flowed through the coil, we have the important relation:

$$\Delta\Phi = -\Delta Q R_L$$

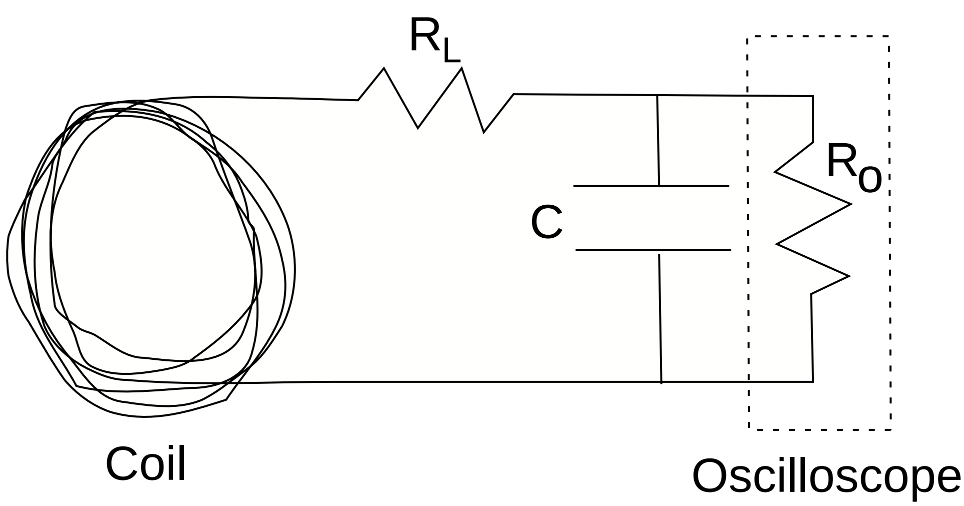

Figure 1: RC Integrator Design.

We shall measure ΔQ by measuring the voltage across a capacitor C in the circuit shown in Fig 1. Unfortunately the input resistance of the oscilloscope R_o isn't usually large enough to be ignored. However, an analysis of this circuit for a completely general $\mathcal{E}(t)$ is really not necessary. If $\mathcal{E}(t)$ is a very short "spike" generated by pulling the coil sufficiently rapidly out of the magnetic field, then immediately after the coil has left the field we have:

$$\Delta V = \Delta Q/C = -\Delta\Phi/R_L C$$

In other words, the oscilloscope's input resistance R_o doesn't affect the charge which flows onto the capacitor. This voltage will then decay exponentially:

$$\Delta V(t) = (\Delta Q/C)\exp(-t/R_p C)$$

where $R_p = (1/R_L + 1/R_o)^{-1}$ is the effective resistance of R_L and R_o in parallel. How fast does one have to pull the coil out of the magnet? Faster than $R_p C$ - the time constant appearing in the exponential above.

Reference H. Zijlstra, *Experimental Methods in Magnetism, Vol. 2: Measurement of Magnetic Quantitites* (North Holland, Amsterdam, 1967), pp. 1-29.

Appendix B: The Hall Effect

Notes by E. A. Schiff, March 1991.

Currents and Drift Velocities

Many properties of electrical conductors can be explained by a very simple model. We imagine that there is a well defined density of mobile charges n inside the conductor. If the charge of each carrier is denoted e, then the total mobile charge in a volume V of the material will be neV. When there is no battery or electromotive force $\mathcal{E}$ applied to the conductor, the carriers move about aimlessly. There will be no average electrical current. When an EMF $\mathcal{E}$ is applied to this material a current does flow, which we can understand by saying that the average velocity of each carrier changes from zero to some finite value v_d (the *drift velocity*). This theory is illustrated in Fig. 2. The current through the rectangle illustrated in the figure is $I = nev_d(hw)$, where hw is the area of the rectangle. A negatively charged particle is illustrated as moving towards the left of the figure. However, the electric field in this case points from the left towards the right of the diagram. The current I also flows from left to right, which is the direction positive charges would move in this circuit.

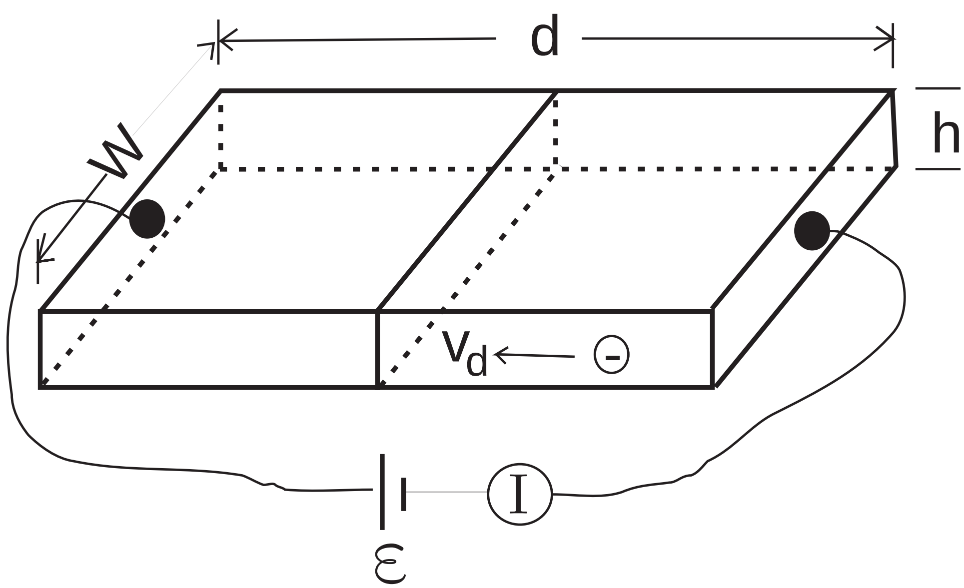

Figure 2: Arrangement for studying currents in a material. An electromotive force $\mathcal{E}$ drives a current I through the material; individual charges (in this case shown as negatively charged) move with an average or *drift* velocity v_d.

If you will think about the theory for a while, you will probably reach the conclusion that it has few obvious consequences which you could test in the laboratory. The first reasonable consequence which occurs to me is Ohm's law: the current through the material is proportional to the electromotive force $\mathcal{E}$ applied to it. $\mathcal{E}$ is applied using a battery or a power supply to *electrodes* at the ends of the specimen as shown in the figure. Now assume that v_d is proportional to the electric field E inside the material:

$$v_d = \mu E$$

μ is called the mobility of the carrier. I shall call the field E the *bias field* across the specimen to distinguish it from the Hall field, which will be described later. Now everything makes sense if the bias field E is constant between the electrodes:

$$E = \mathcal{E}/d$$

First, a constant field of this size is consistent with the electromotive force $\mathcal{E}$ across the specimen. Second, the drift velocity is constant everywhere, so the current crossing any rectangle between the electrodes is a constant. So Ohm's law really means that the drift velocity v_d should be proportional to the field. It may surprise you at first that this is the explanation for Ohm's law: perhaps you thought that an electron should accelerate in an electric field! I'll return to this question later.

The Hall Effect

In 18?? E. E. Hall at Johns Hopkins University discovered another extremely important consequence of the theory. If a magnetic field $\vec{B}$ is applied perpendicular to $\vec{v}_d$, there should be a magnetic force $\vec{F}_L$ on the particle according to the Lorentz force law: $\vec{F}_L = q\vec{v} \times \vec{B}$. This force is illustrated at the bottom of Fig. 3, where it is shown as being perpendicular both to the magnetic field and the drift velocity. Pay careful attention to the direction illustrated for $\vec{F}_L$: do you agree with the direction in which it has been drawn?

The consequences of this Lorentz force are not immediately obvious. Some carriers might be expected to leave the bottom of the conductor. Clearly, there must be some kind of "box" which holds the carriers inside - if there weren't, carriers would have poured out of the conductor whether or not a magnetic field was applied. Since we are assuming that negative charges carry the charge, we have illustrated a negative charge built up on the bottom surface of the conductor. Since the slab as a whole is still uncharged, there must be an exactly equal buildup of positive charge on the top of the slab as well. These charges create an electric field perpendicular to both the drift velocity and the magnetic field; this electric field is called the *Hall field* E_H. The Hall field builds up until the electrostatic Hall force F_H exactly cancels the Lorentz

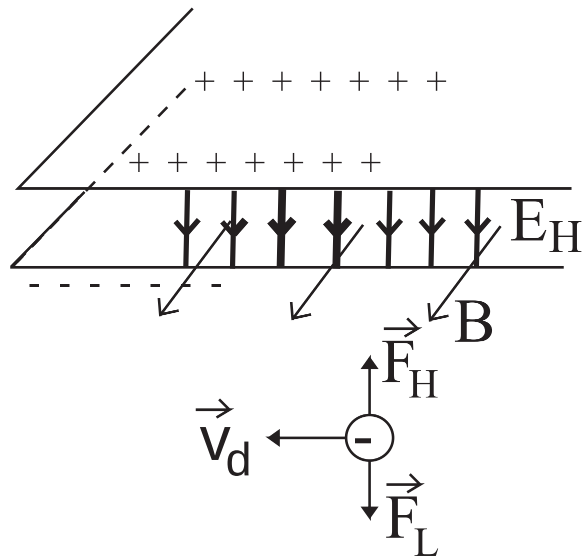

Figure 3: The Hall effect. A magnetic field is applied perpendicular to the direction of current flow. The Lorentz force $\vec{F}_L$ is cancelled because it causes charge to build up on the surfaces of the material. The resulting electric force $\vec{F}_H$ exactly cancels the Lorentz force; the extra electric field $\vec{E}_H$ is measurable.

force: $\vec{F}_L + \vec{F}_H = 0$. Then there is no net force on carriers which is perpendicular to their drift velocity.

If we substitute the expressions for the Lorentz force due to $\vec{B}$ and the electric force due to $\vec{E}_H$ into this force balance equation, we obtain:

$$q\vec{E}_H + q\vec{v}_d \times \vec{B} = 0.$$

The charge q can be cancelled on both sides of this equation, obtaining a simpler relation:

$$\vec{E}_H = \vec{v}_d \times \vec{B}.$$

The magnitudes of these vectors are simply related when, as illustrated in Fig. 3, $\vec{B}$ is perpendicular to $\vec{v}_d$:

$$E_H = v_d B$$

where the equations now refer to the magnitudes of the various lengths involved.

Hall's theory makes two testable predictions. First, at a given current and drift velocity inside the material, the Hall field should be proportional to the magnetic field. Second, the Hall field should be proportional to the electrical current being drawn through the specimen, since this current is proportional to the drift velocity.
Hall Effect Measurements

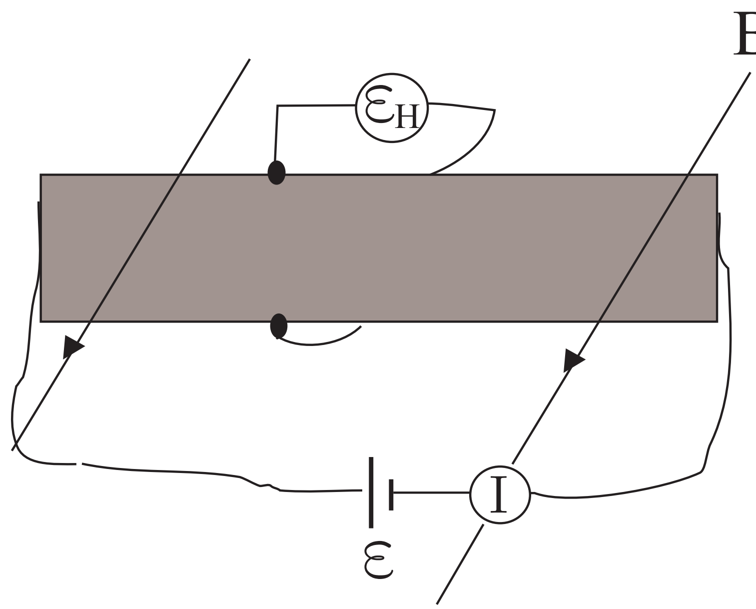

Figure 4: Arrangement for studying the Hall effect. A current is driven through the specimen by the electromotive force $\mathcal{E}$. A magnetic field is applied perpendicular to the direction of the current flow. The resulting Hall voltage $\mathcal{E}_{\mathcal{H}}$ can be measured with a standard voltmeter.

To test the theory, consider the arrangement in Fig. 4. A battery with electromotive force $\mathcal{E}$ is connected to the ends of the sample; an ammeter is provided to measure the current I through the specimen. A magnetic field is applied perpendicular to the current flow as shown. Two tiny electrodes are attached to the top and bottom of the sample to measure the electromotive force $\mathcal{E}_H$ perpendicular to the direction of the current flow. For a specimen of height h (see Fig. 1) the Hall field and the electromotive force $\mathcal{E}_H$ are related according to:

$$E_H = \mathcal{E}_H / h.$$

The first test we suggested earlier was to verify that the Hall field E_H is proportional to the magnetic field: $E_H = v_d B$. If this is true, the constant of proportionality is the drift velocity, and Hall's wonderful theory gives us our first information about how fast the carriers are drifting inside the material. This is not all: the sign of the Hall voltage tells us in which direction the carriers are drifting, and can be used to infer the charge of the mobile carriers. In most materials, Hall discovered that the sign of the charge is negative - thus showing that electrons carry the charge. This sounds great, but occasionally the sign of the charge is positive. This puzzle needed over half a century until a theory of electrical currents carried by *holes* explained the effect.

The second test is that the drift velocity should be proportional to the current flowing through the specimen: $I = (hw)nqv_d$. It is best to express this relation in terms of a *current density* $j = I/(hw)$, where hw is the area of the rectangle drawn in Fig. 2:

$$j = nqv_d$$

The current density is the current per unit area through the specimen, and has the dimensions of Amps/m^2. If the current density and the drift velocity are measured to be proportional, then we have learned the value of nq inside the material. Of course Hall couldn't have guessed what the size of each particle's charge might be. However, if we think that the charge of the carriers is the same as that for an electron ($q = 1.6x10^{-19}$Coulombs, then the density of charges n can be calculated given the current i and the drift-velocity v_d:

$$n = j/qv_d$$

Finally, we return to Ohm's law: $v_d = \mu E$. We graph the drift velocity v_d measured using Hall's method versus the electric field $E = \mathcal{E}/d$ due to the battery or power supply. If a proportionality is obtained, we have measured the mobility μ of the carriers. Ohm's law is obeyed by most materials in most experiments, and we return to the question of why the velocity is proportional to the electric field at all. One can make a satisfactory analogy of an electric force on a carrier to the gravitational force on a ball. Motion of the carrier through a material seems to be more like dropping a ball through a viscous oil than like dropping it through air. In air the ball steadily accelerates. In oil the ball only briefly accelerates, and then reaches a constant *terminal velocity.* The drift velocity measured in solids is apparently the terminal velocity of the carrier inside the material when acted on by an electric force.
Endnotes

The geometry shown in Fig. 3 is the simplest which can be drawn for illustrating Hall's effects, but the Hall effect is usually measured with other specimen shapes. The arrangement of charges which leads to the Hall voltage is much more complicated for these shapes, but the equations and arguments remain valid: the Hall field $\vec{E}_H$ is constant throughout the material if the electric field $\vec{E}$ driving the current flow is constant.

A serious treatment of the Hall effect can be found in K. Seeger, *Semiconductor Physics* (Springer, Vienna, 1988).