

MEASUREMENT OF THE MUON LIFETIME

PURPOSE

- Learn about cosmic rays
- Understand the concept of a particles lifetime
- Apply statistical analyses to a real problem

1) Introduction

Interstellar particles that impinge upon the earth are called “cosmic rays.” Such particles must be stable in order to travel the enormous distances from either within our galaxy or other galaxies to reach the Earth. The only truly stable particles we know are protons, electrons, neutrinos (and their antiparticles) and photons. Most of the primary cosmic rays are indeed protons, but there are also some stable nuclei.

Cosmic rays have been studied extensively starting in the 1920’s and studies still continue today. Several particles have been discovered in cosmic rays. The carrier of the strong force, called the pion (π) was predicted by Yukawa in the 1930’s to be about 100 MeV in mass. It was believed to have been discovered in cosmic rays by J C Street and E C Stevenson at Harvard University and almost simultaneously at Caltech by C D Anderson and S H Neddermeyer in 1937. However, they had not discovered the pion, but had found an unexpected particle of about the same mass that we call the muon (μ^\pm). The muon does not have any strong interactions, but behaves exactly like an electron, although its mass is 105 MeV, about 200 times larger than the electron. There isn’t a really good explanation of why nature has muons. When the famous American physicist I. I. Rabi heard of the muon, he said “Who ordered that?” Rabi would be even more perplexed today as yet another generation of electrons, called taus (τ^\pm) have been discovered.

To produce muons, primary cosmic ray protons interact in the upper atmosphere and produce pions. These pions with a mass 273 times that of the electron are not stable. They decay “radioactively” into muons. The muons that are so produced, due to their longer lifetime and almost complete absence of nuclear interactions, are the principle components of penetrating particles produced by cosmic rays that are observed at sea level; the sequence of events is illustrated in figure 1.

The particles arriving from space are known as primary cosmic rays whereas the particles created in the collisions are known as secondaries. Many of the new particles are very short lived and do not survive to reach sea level, but positive and negative pions created in the process decay into muons that are detectable at ground level. The total secondary flux at sea level is about $1 \text{ cm}^{-2} \text{ min}^{-1}$. (See <http://pdg.lbl.gov> for more information.) Roughly 75% of the flux consists of positive and negative muons, 25% of it consists of electrons and positrons. If the negative muons stop in matter then they can be captured by atoms whereas the positive ones remain free. The mean lifetime of both

positive and negative free muons is a few μs (10^{-6} sec). Some μ^- can be captured by atoms and these have a slightly shorter lifetime.

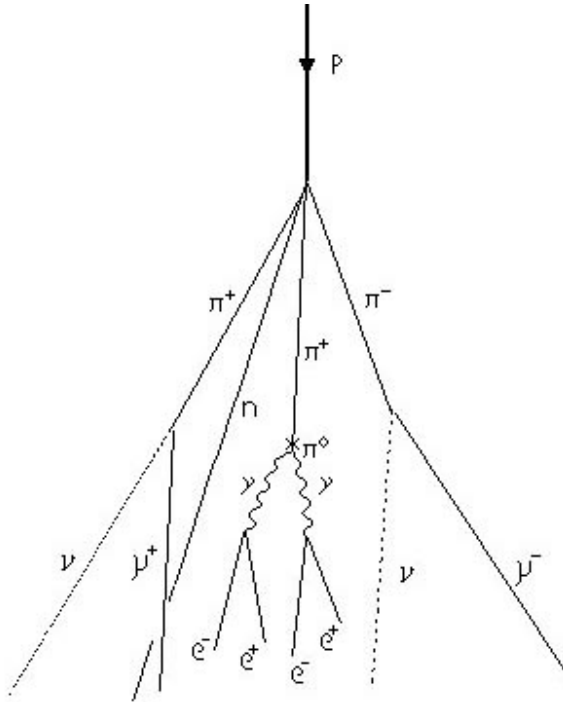


Figure 1. A typical interaction between a cosmic ray proton and an atmospheric nucleus

According to classical physics, if the mean lifetime of the free muons is a few μs then they should only travel a few hundred meters when traveling at the speed of light after being created in the upper atmosphere and many fewer than that indicated above would be expected to reach the ground. The only way to explain the large number of muons detected at sea level is through relativistic time dilation.

Muons continue to be of interest in many areas of physics, including precision studies of its magnetic moment ($g-2$), in investigating the structure of materials using spin-relaxation techniques.

2) Radioactive Decay

The physical mechanism that causes both pions and muons to be unstable is described by the theory of the weak interactions. It is quite natural that particles, states of nature, decay. In fact, particles are stable only if some rule forbids them to decay. For example, if there are no other particles of lower mass for them to decay into.

Consider any system of N particles where the probability to decay (or transform in any manner) is proportional to the total number present. Call the number that decay in a short interval dN . Then

$$dN \propto N \text{ or}$$

$$dN = cN, \text{ where } c \text{ is an arbitrary constant}$$

$$\frac{dN}{N} = c$$

Integrating both sides of the equation from time to t_0 time t , where the starting population is N_0 at time t_0 and N at time t gives

$$\int_{t_0}^t \frac{dN}{N} = \int_{t_0}^t -c$$

$$\ln[N(t)] - \ln[N(t_0)] = -c(t-t_0)$$

$$\ln\left[\frac{N(t)}{N(t_0)}\right] = -c(t-t_0)$$

$$\frac{N(t)}{N(t_0)} = e^{-c(t-t_0)}$$

Let us now take $t_0=0$. We then have

$$N(t) = N_0(t)e^{-ct}$$

Identifying the lifetime (call it τ) as $1/c$, we can write the master equation for radioactive decay as

$$N(t) = N_0(t)e^{-t/\tau}$$

This gives an exponential distribution. *Plot this curve where the horizontal axis (the ordinate) is in units of lifetime τ . You should calculate the fraction of particles that decay in one lifetime, in two lifetimes and in three lifetimes.*

3) Experimental Method For Determining the Muon Lifetime

Muons decay via the weak decay process:



where the ν 's are neutrinos. The subscripts indicate if they are electron like or muon like and the bars indicate particle or antiparticle.

We use a tank filled with liquid scintillator. This material has chemicals in it that are transparent and emit light when a charged track passes through. There are two photomultiplier tubes placed at the top and bottom of the tank. It is possible, even usual, to do this experiment with only one such tube, so lets concentrate on that for now. The photomultiplier tube is capable of sensing photons produced by the liquid scintillator. It consists of a glass window, the inside of which is coated with a chemical mixture that is efficient at converting single photons to electrons (this part is called the photocathode, since it acts as a cathode). After the "photoelectron" is produced it is accelerated by an electric field until it hits another structure called a dynode. There it smashes into a metallic surface and produces more electrons. Photomultiplier tubes have several stages, usually 6 to 12. The tubes we are using have 12 stages. High Voltage is used to accelerate the electrons between each dynode. This is usually supplied with one input and then the voltage is divided by a unit that is called the "base," because it plugs into the phototube. The typical conversion efficiency, the fraction of photons that make electrons, depends on the wavelength of the photon, but is around 16%. (This is usually called the "quantum efficiency.") It is made up of the efficiency of the photocathode and the collection efficiency of the first dynode. Many photons are produced when a track crosses liquid scintillator. The number collected depends on several factors: the number produced, the collection efficiency and the quantum efficiency of the phototube.

Many muons pass through the liquid scintillator producing light and signals in the photomultiplier. We are not interested in the ones that traverse the tank. Occasionally one of them stops in the tank. Basically it gets slowed down by losing energy due to ionization and comes to rest. Subsequent it

decays as discussed above. From these stopping muons we get two pulses: one from the muon and a second one a short time later from the electron arising from the muon decay. The time delay between the two signals is measured for a large number of decays and produces an exponential distribution. Now this is key-it doesn't matter when you start the clock on an exponential decay, to get the lifetime all you need do is measure the decay constant which on a logarithmic scale is just 1/slope of the line.

We use two phototubes and make sure that they both produce a signal at the same time (this is called in "coincidence"). The reason to do this is that phototubes can produce a signal when there is none because an electron is spontaneously emitted from the photocathode or one of the early dynode stages. The phototube signal is typically quite short, ~ 10 ns. The output from the PMT is passed to a discriminator, an electrical circuit that fires if a signal is above a preset level. If a signal is above the minimum level a logic pulse is output with an adjustable width. The discriminated logic pulses are sent to another electrical circuit called a coincidence unit. This unit produces a logic pulse if signals from both PMTs are present at the same time. (Descriptions of some of these circuits are available at <http://www-esd.fnal.gov/PREP/>). Output of the coincidence unit is sent to a computer (PC) that starts a counter, with counting a rate of 10 MHz, implying a resolution of $0.1 \mu\text{s}$. If a second pulse (from the e^+) arrives after the first, the counter is stopped and the PC signaled to indicate that the counter has stopped and is ready to be read. The PC records the time interval and resets the electronics so it is ready for more decays. If a second pulse does not arrive, the counter continues until a count of $25.5 \mu\text{s}$ is reached. Then the circuit resets itself. It is possible to achieve an error of about $\pm 0.04 \mu\text{s}$ with a few days of running.

4) Procedure

4.1 Apparatus:

Basic components of the experiment are one tank (filled with liquid scintillator), two photomultiplier (PMT)s with electronics attached to them and a computer with ADC board. The diagram shown in figure 2.

Also you need one multi-channel discriminator (MODEL 321B); one dual 3 fold logic unit (MODEL 162); one converter (the white unit in the middle of the equipment set used to convert the signal into a 10 mV TTL level), one scalar (ORTEC 770), if you want to monitor the counting rate independent of the computer, and one interface board (NI, CB-68LP), used to transfer the signals to the computer.

4.2 Operation:

Before you start, make sure all equipment is off.

Safety considerations:

- Be careful of the high voltage (the two red wires).
- Make sure your connections (especially the high voltage value) are correct to avoid damaging the photomultiplier tubes. *Before you turn on any high voltage ask the instructor to approve your setup.*

(a) Set the High Voltage of the photomultipliers:

There are two wires connected to each photomultiplier, the red one is for high voltage and the black one is for the signal.

- The voltage of the top PMT should be set as NEGATIVE 2500 V. Key in the number on the digital switches on the front of the power supply. (The actual voltage is 2600 V);

- The voltage of the top PMT should be set as NEGATIVE 2750 V. Key in the number on the digital switches on the front of the power supply. (The actual voltage is 2900 V);
- There is a spare PMT “B1” attached to a scintillation counter for other tests. If you use the PMT B1 its voltage is POSITIVE 1900 V (digital number).

(b) Connect the signal wire of the PMT to discriminator (model 321B, quad discriminator), choose inputs labeled 1, 2 or 4 to connect (3 is broken).

(c) Connect the corresponding output of the discriminator to the DUAL 3 FOLD LOGIC UNIT. (Actually we will only use two of the inputs, so we could have used a 2 Fold unit, but this is what we had available.) To record the coincidence of top and bottom PMT, connect the two signals to DUAL 3 FOLD LOGIC UNIT input. Only one output signal from the logic unit, and input to the converter (the white unit, in the middle of the equipment set). Then connect the output to the interface board (green card on the table, which is connected to the computer). Make sure you do not touch the connections on the interface board. On the interface of the discriminator and the logic units, you need to terminate with 50 Ω the extra input and output connectors to make sure the signals are not influenced by them.

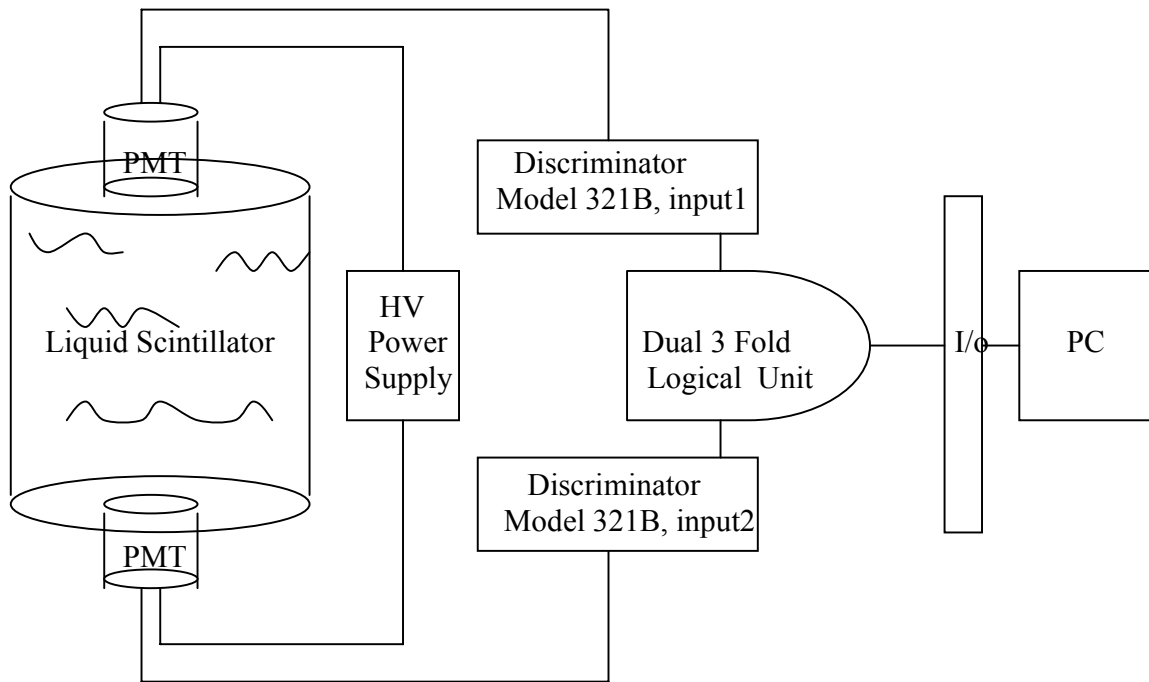


Figure 2. Schematic plot of the muon lifetime experiment

(d) Make sure the connections are correct, *check with the instructor*, and then turn on the power to the PMTs.

(e) Turn on the computer and on the screen, open (double click) the file named "Muon Lifetime." This program is written in the “Lab View” computer language and is a modern program used widely to acquire data. You will see two data collection windows. Set the timeout (μs) to 20 on the leftmost window. This window will display the difference between start and stop times when the is delay < 20

μ s. You can change the vertical scale during the data collection if you like. The right window will just indicate single signals. Then click the “Run” arrow on the top-left of the window to collect the data. You will be asked to specify a file to save the data. You should name this file. If you happen to stop the run at any time, the file will be appended to if you start up again and give the same file name.

(f) After you finish recording the data, on the file you saved and you will see a column of data. They are the different time intervals for each muon decay. These data must be separated into bins of equal time units. This is called a histogram. The time interval will depend on how much data you have recorded, since we want to make sure that all bins have at a few events.

To make this histogram, you can use the EXCEL program. Read in the data into a column, then generate another column that gives the decay time values at the left edge of the bin. Select the “tools” menu, select “data analysis” and click on “histogram.” This will generate a two columns, one being the time difference bins and the other the number of events in these bins. To see the histogram you need to check the “chart” box in the dialog that appears when you make the histogram. At this point select on the numbers only in the two columns and copy them to the clipboard. Ignore the titles and the overflow or “more” row.

(g) Calculate the mean lifetime:

We will find the mean lifetime in several ways using the “SigmaPlot” program. You’re your data into the first two columns of the data sheet. Column 1 should have the time difference values and column 2 the number of events. One method is use the fact that the \ln of an exponential is a straight line, and in this case $1/\text{slope}$ of the line is the lifetime. To find the slope we can plot the $\ln(\# \text{ of events})$ vs time. To do use the transform function in SigmaPlot. If you pasted the time data in column 1 and the number of events in column 2, then you select “transform” and type “col(3)=ln(col(2))”. This puts the \ln of the number of events in column 3. Now its not enough to just look at the numbers, we also need the errors. This can be accomplished by evaluating the error on the number in each bin. For example, if you measured N events the error is the \sqrt{N} . Thus you can make two new columns using the transform function one being the $\ln(N+\sqrt{N})$ and the other $\ln(N-\sqrt{N})$. One column gives the upper error and the other the lower error at the level of one standard deviation (or 1σ , see D. C. Baird below for a discussion). Graph the three columns versus the time difference using the Sigma Plot function. Print out the plot and with a ruler estimate the slope and by varying within the error bars, estimate the error on the slope. Translate this to a value for the muon lifetime and the error. Show this value to the instructor.

We will now use some sophisticated linear regression tools to obtain the value of the lifetime and the statistical error. Here we can do a “least squares fit” to the exponential distribution. The idea is that we can minimize the differences between the hypothesis of an exponential distribution and the data points and this minimization gives the best value of the lifetime. Here you select “Statistics” then “Regression Wizard” specify in “Equation Category” Exponential Decay and in “Equation Name” Single, 2 Parameter. The program will give you back the value of $1/\tau$ and the error on $1/\tau$. Convert these to the lifetime and its statistical error.

(h) List the possible sources of systematic error. Then make an estimate of the systematic error in your measurement.

(i) Options:

1. If you want to see the count of one PMTs, just connect the signal to the 770 COUNTER (right side of the unit) and push count/stop to find a counting rate.
2. You might want to check the relative timing of the two phototubes. The signals from the phototubes come along a 50 Ω cable with a speed of about 1 ft/ns. The pulses may not be arriving at the coincidence unit at the same time. You can check this using the oscilloscope, just trigger on the pulse from one tube and see where the second tube signal is relative to the trigger. You can adjust the timing by adding or reducing the cable lengths.
3. You might want to insure that the discriminator output widths are sufficiently wide to take into account any jitter in arrival times of the phototube pulses, from for example where in the tank the muon decays.
4. In all real experiments there are many other data collection modes that are designed to specifically test for backgrounds or other systematic effects. You might want to try evaluating some of these by trying some of the next tests or others that you may devise. For example:
 - Take data using only one phototube.
 - Put the bottom plastic scintillation counter B1 in coincidence with the ones on the tank. In this case, we do not expect to any useful muon decay data, because the decay electron should not hit B1. So this is one way to measure the background.

5) Further Analyses

- (a) Report the value of the muon lifetime, the statistical error and the systematic error. Include a discussion of any checks you did to verify the systematic error. Discuss how the experiment could be improved.
- (b) Using your value for the lifetime estimate the mean momentum muons must have in order to reach the Earth's surface, about 15 km below the place in the atmosphere where muons are created. Here you must take into account relativistic time dilation.
- (c) Why don't you have to worry about relativistic time dilation in analyzing your data?

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