

## Introductory Measurement Procedures and Error Analysis

### **Introduction:**

A measurement of any physical quantity is not complete unless the experimenter specifies his/her uncertainties in performing the measurement. Such error analysis is crucial to give the experimental results meaning, so that they can be compared with theoretical predictions or related measurements. Fortunately there are standardized procedures for estimating measurement uncertainties and propagating these errors through various calculations. In this introductory project for **Phy 344**, you will work on several exercises involving simple measurements and learn these error analysis procedures that will be important for the rest of the experiments you will perform in the lab this semester, as well as throughout your future scientific career!

This project should be possible to complete after the first few class periods. While working on these exercises, you will learn the general procedure for maintaining a lab notebook, which should include sketches, descriptions, tables of data, calculations, plots, and a discussion of your results. Please see the course syllabus for more details about the lab notebooks. You will also become familiar with the Origin plotting and analysis software that will be useful for many of the experiments you will perform throughout the semester. For this project you will submit your lab notebook only and will not be expected to turn in a separate lab report.

**Required Reading:** Read chapters 1-5 and chapter 8 of the Taylor error analysis textbook.

### **Exercises:**

#### **A. Measuring Disks**

In this exercise, you will measure the circumference and diameter for several disks and estimate the associated uncertainties. Following the procedures detailed in the textbook, you will propagate the error through a calculation of the ratio of the circumference to the diameter for each disk.

1. Choose five disks from the supply, covering a wide range of sizes. Make a table in your notebook with columns for diameter, circumference, the associated measurement uncertainties, and two final columns for the ratio of the circumference to the diameter and its uncertainty.

2. Measure the diameter of each disk and estimate your uncertainty, both in resolving the scale on the ruler and in ensuring that the diameter is properly defined.
3. Devise a technique for measuring the circumference of each disk, perhaps making a flexible measuring tape from a strip of paper. Consider possible sources of error in this measurement and estimate the uncertainty.
4. For each disk, compute the ratio of the circumference to the diameter.
5. Using the appropriate technique for propagating errors, determine the uncertainty in these ratios.
6. After measuring all five disks, compute the mean of your distribution for the ratio of circumference to diameter and the standard deviation of the mean. This can be done easily with the statistics functions in Origin. Review Chapter 4 of Taylor, particularly section 4.5, for guidance.
7. Using your measured data, plot circumference vs. diameter for each disk, including error bars based on your estimated uncertainties. Determine the best fit to a linear dependence using Origin along with the error in the best fit line.
8. Compare these values, both from step 6 and step 7 along with their associated uncertainties, to  $\pi$ .
9. Describe possible sources of systematic error in this measurement and discuss possible methods for quantifying and reducing these.

## **B. Oscillations of a Pendulum**

For a simple pendulum with length  $L$ , recall that the period is given by  $T = 2\pi\sqrt{L/g}$ . By measuring  $T$  and  $L$ , while keeping track of uncertainties, you will compute a value for  $g$ . You will also explore the possibility of using repeated measurements to reduce the uncertainty in a particular quantity.

1. Arrange a pendulum at your lab bench by attaching a clamp to the edge of the table and selecting a string and bob. Obtain a timer and get a feel for its stopping and starting mechanism.
2. Measure the length of the pendulum and record your measurement uncertainty. Consider carefully how the length should be defined.
3. Start the pendulum oscillating with an amplitude that is not too large. Consider the consequences of a large amplitude swing and discuss this in your notebook.

4. Using a stopwatch, record the period for one oscillation of the pendulum. Repeat this measurement ten times. Compute the mean, standard deviation, and standard deviation of the mean for these measurements. What should you quote for your best measurement value and associated uncertainty for this period? What is the dominant source of the measurement error?
5. Use the stopwatch to measure the time for the pendulum to complete ten oscillations. Determine the period and the uncertainty associated with this measurement. Repeat this procedure ten times – so, ten trials, each trial consisting of ten oscillations of the pendulum. Perform the same statistical analysis on your data as in step 4. Compare your uncertainty in this measurement of the pendulum period with that from step 4. Discuss potential problems with this measurement approach.
6. Use your measurements of the pendulum period and the length to determine your best value for the acceleration due to gravity near the surface of the earth. Compare this with the accepted value for  $g$ .
7. Change the length of the string on your pendulum and measure the period using the multiple oscillation method from step 5. Assuming your uncertainty from step 5 is reasonable, you may take this value for the uncertainty in the new measurement of the period without repeating multiple measurements.
8. Adjust the length of the pendulum two more times and measure the period for each so that you obtain four data points of length and period. Plot these data along with error bars in Origin in such a way that a linear fit will have a slope which can be compared with  $g$ .
9. Describe possible sources of systematic error in this measurement and discuss possible methods for quantifying and reducing these.

### **C. Measurements of Height with a Quadrant**

The quadrant is a simple device for measuring height by sighting to a particular point and measuring the angle above the horizontal. This will provide another situation for practicing estimating uncertainties and propagating errors, including techniques for dealing with a nonlinear function (review section 3.5 from Taylor).

1. In order to translate the angular measurement from the quadrant into a height, you will also need to measure the distance from your observation

point to the base of the object being studied. This can be done by measuring the size of your stride, then pacing off the distance. Devise a procedure for characterizing your uncertainty in this measurement. In addition, you will need to measure the height of your observation point – estimate the uncertainty in this measurement.

2. Draw a sketch in your notebook of the relevant geometry for such a height measurement using a quadrant, labeling the various lengths and angles.
3. Discuss possible sources of error in the reading of the angle on the quadrant and estimate your measurement uncertainty for this quantity.
4. Use your quadrant to measure the height of the ceiling in the lab room (Room 377), that is, the distance from the floor to the ceiling. Propagate the errors to determine the uncertainty in your calculation of the ceiling height and compare your calculated value to a direct measurement using a ruler.
5. Using the same technique, go outside and measure the height of the Physics Building along with the associated uncertainty.
6. Describe possible sources of systematic error in this measurement and discuss possible methods for quantifying and reducing these.

#### **D. Measuring a Distribution of Resistances**

In this exercise, you will measure many nominally identical resistors then study the distribution of the measured resistances. Review Chapter 5 in Taylor.

1. Choose a bag of nominally identical resistors from the supply table with labeled values of at least  $1\text{ k}\Omega$ .
2. Measure the resistance of each resistor with an ohmmeter and record the value in your notebook. Repeat this for at least 50 resistors with the same labeled resistance values.
3. Plot these resistance values in a histogram using Origin. Explore the effects of choosing different bin sizes.
4. Compute the mean of this distribution. There is a straightforward way to do this with Origin – ask a TA or the instructor for help if necessary.
5. Compute the standard deviation, again, using the statistics tools in Origin.
6. Compute the standard deviation of the mean. Using this, compare the mean value of your distribution with the labeled value on the package of resistors.