Analysis of Gravitation

Notes by E. A. Schiff, March 3, 1992 & B.L.T. Plourde, 2009.

Introduction:

Gravitational forces are part of everyday life, but the large magnitude of this force is a consequence of the large mass of the Earth. One has very little experience with the gravitational forces between objects of ordinary size. However, measuring these forces is required to estimate the "universal" gravitational constant G proposed by Newton. This constant is essentially a proportionality factor between gravitational force and the masses of objects. To determine the constant requires a system in which both the masses of the objects as well as the gravitational forces are directly measured. We can only directly determine masses of objects of ordinary size, since we need to apply a force and measure response to estimate mass from Newton's law $F = ma$. Thus we must measure gravitational forces between ordinary objects to determine G.

Newton himself did not know the magnitude of G. Some years passed after Newton's description of the theory of gravitation before Cavendish succeeded in determining it by detecting gravitational forces between lead spheres. When Cavendish had succeeded, he said that he had "weighed the Earth," since knowledge of the gravitational constant, the gravitational acceleration at the Earth's surface, and the Earth's radius permits the mass of the Earth itself to be estimated.

In this experiment you will use a modern recreation of the Cavendish apparatus to measure the universal gravitational constant. The apparatus is based on a torsional pendulum. A pair of weights on a hanger is hung on a slender wire; if the hanger is twisted slightly, it oscillates rather slowly back and forth. This same device can be used as a very sensitive balance, and in this experiment the balance will be used to detect the small gravitational force due to two larger lead balls placed close to the weights on the hanger.

Objectives

- Learn the design, alignment, and calibration of an optical system for monitoring the rotational motion of a mirror.
- Measure the motion of a damped torsional oscillator; this oscillator is used as a highly sensitive torque balance.
- Learn the theory of damped simple harmonic oscillation.
- Compare the torsion oscillator measurements with the theory of the simple harmonic oscillator.

- Measure a gravitational torque using the balance, and estimate the universal gravitational constant G based on these measurements.

Suggested References:

1. Instruction Sheet for gravitational torsion balance from Leybold. Contains detailed specifications for the apparatus, including its alignment and operation.

2. H. F. Meiners, W. Eppenstein, K. H. Moore, *Laboratory Physics* (John Wiley aand Sons, Inc., New York, 1969), pp. 182-189. Instructions for several different experiments with the apparatus. If time permits you are supposed to execute the full project called "deflection method with error analysis."

3. Textbook descriptions of a "torsional" oscillator and of a damped simple harmonic oscillator. *Torsional oscillator:* David Halliday and Robert Resnick, *Fundamentals of Physics, Third Edition* (John Wiley and Sons, New York, 1988), pp. 312-313. *Damped oscillator:* Halliday and Resnick, pp. 318-319. *Damped oscillations and resonance phenomena:* Iain Mains, *Vibrations and Waves in Physics* (typically on the bench with the Torsion Pendulum experiment).

4. Read about the search for gravitational waves at LIGO, the Laser Interferometer Gravitational-Wave Observatory, on the LIGO website – http://www.ligo.caltech.edu/

5. Optional - An interesting alternative to the technique you will use was described by Michael S. Saulnier and David Frisch in *Am. J. Phys.* **57**, 417 (1989).

Suggested Apparatus:

1. Gravitation Torsion Balance. Leybold-Heraeus, Inc. torsion balance (Klinger catalog KM1115). Includes mounting and two lead spheres (about 1.5 kg each). Copies of manual from Leybold should be on lab bench.

2. Laser pointer with mount and switch. No manual.

3. Glass ruler and mounting. No manual.

4. Spirit level. No manual.

5. Meter stick and triangle.

6. Bobs and string for alignment.

Safety Considerations: Avoid dropping the lead spheres on your toes.

Avoiding Damage: The torsion balance must not be moved if a special clamping device is not engaged. The filament that suspends the pendulum is quite delicate and somewhat difficult to replace if broken, so, please treat the balance assembly with care.

Instructions - Theory of Torsional Oscillator

- 1.A damped torsional oscillator is usually assumed to obey the following equation:

$$I\frac{d^2}{dt^2}\theta(t) = -\kappa\theta(t) - D\frac{d}{dt}\theta(t) \tag{1}$$

  where $\theta(t)$ describes the angular position of the oscillator, I is its moment of intertia, κ is a torsion constant, and D is a damping constant. If the damping constant D is small enough, this equation is solved by functions such as

$$\theta(t) = \theta_0 \cos(\omega t)e^{-kt} \quad . \tag{2}$$

  Make sure you understand where these equations come from. Show that this function solves the equation given earlier, and determine the values of ω and of k in terms of κ, D, and I.

- 2.In this experiment you will *calculate* the moment of inertia of the torsional oscillator. Study the construction of the torsion balance, and determine the formula for its moment of inertia I. Compute the moment of inertia for the torsion oscillator. Approximately how large an error do you expect for your estimate of I? Note that some of the parameters of the apparatus, e.g., the mass of the small Pb spheres, could only be measured by dismantling the balance – **do not do this!**. Discuss this with the instructor or TA and use the previously measured values for such quantities.

Instructions - Design of Balance Optics

- 1.There is a mirror attached to the hanger of the torsion balance. As the balance swings, you can use a light beam reflected from this mirror to monitor the angular displacement of the torsion balance.

  The light beam itself originates at the laser pointer with a small wire stretched across its output. The beam is directed at the mirror, and an image of the beam interrupted by the wire is reflected onto a ruler.

Make a drawing of the optics, including the laser pointer, torsion balance, and the ruler. Make some notes to explain how you will place and align the the illuminator and the ruler so that a clear image of the beam from the pointer is obtained on the ruler.

- 2.Derive an expression relating the position of the image on the ruler to the angular position of the torsion oscillator.

Instructions - Alignment and Calibration of Balance

1. Set up the balance and unclamp it. Carefully align the torsion balance so that the aluminum hanger does not touch the plastic guides.

2. Set up the illuminator and ruler so that the mirror in the torsion oscillator is illuminated, and the reflected beam is imaged on the ruler. There are several possible adjustments: the location of the illuminator, the condenser lens on the illuminator, the location of the ruler, and their relative orientations. Record your alignment procedure, and the configuration of the illuminator, balance, and ruler. You will need to mark the locations of the illuminator and ruler in case they are accidentally moved.

3. The most serious problem in aligning the apparatus is arranging for the small lead spheres on the torsional oscillator to have a "rest" position which does not touch the housing. There is an adjusting screw at the top of the apparatus for this alignment. Consult an instructor if you need to make any adjustments, since it typically requires several hours to do this.

4. Place the large lead balls onto their mounts, and rotate the hanger so that the two balls nearly touch the glass of the housing.

5. Start the oscillator moving by *gently* touching the oscillator with the clamping device on the bottom. Set up a graph to record the position of the wire image at time intervals of 30 s. Record the position of the wire image on the ruler every 30 seconds until the oscillator has completed two full cycles or more; this takes at least 30 minutes.

Instructions - Analysis of Balance Motions

- 1.Plot your measured oscillations in Origin and make a fit to a damped sinusoid. Determine the oscillation period T of the balance, and estimate the error in this period. Calculate the torque constant κ of the balance based on this estimate, and estimate the error in the torque constant.

- 2.Assess whether your measurements have the same form as predicted by the theory of the damped simple harmonic oscillator.

Instructions - Gravitational Torque Measurements

- 1.Allow the apparatus to come to an "equilibrium" position; there are no noticeable oscillations when the apparatus is in equilibrium. Record readings of the wire image for several minutes to prove that equilibrium has been achieved and to estimate the errors in your measurements. Then "reverse" the large lead spheres by gently rotating the mount until the spheres nearly touch the housing on the opposite side. The torsion balance should lose its equilibrium because of the change in the gravitational forces exerted by the spheres on the balance. Measure the angular displacement of the balance as a function of time following this reversal. Ideally you should continue measuring until a new equilibrium position is established; this will take more than an hour.

- 2.Plot your measured oscillations for this new configuration in Origin and make a fit to a damped sinusoid.

- 3.Then return the spheres to their original position. Make occasional measurements of the oscillator until equilibrium is reached. This step is only performed to determine how reproducible your original measurement of equilibrium position is after about two hours; for this reason you should try to complete steps 1. and 2. in one laboratory period.

Instructions - Analysis of Gravitational Torques

- 1.Analyze your data to estimate the gravitational constant from the change in equilibrium position of the torsional oscillator. Use your damped sinusoid fits for the two extreme configurations of the large Pb spheres. Compare with the textbook value.

- 2.Analyze the errors in your measurements. Is the deviation between your measured value of G and the accepted value consistent with this analysis? What is the principal source of error in this measurement?

Memoranda

- •Can you explain why there is an image of the wire in front of the pointer on the ruler?

- •Did the $\theta(t)$ data you obtained agree with the theory of the damped torsional oscillator?
- •Were you able to account for the discrepancy between your estimate of G and the "book" value by random measurement errors?
- •Did you note any "systematic errors" in your procedure for analyzing the torsion balance data?
- •Can you explain why Cavendish said he had "weighed the Earth" when he completed his measurements?