Quantum Field Theory II

Problem Set 2

Prof. C. Armendariz-Picon

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Exercise 4: Projecting onto the out-vacuum

Show that

$$\lim_{t_f \to (1-i\epsilon)\infty} \langle \vec{\phi}_f, t_f | = \langle \vec{\phi}_f | 0_{\text{out}} \rangle \, e^{-iE_0^{\text{out}}(t_f - t_{\text{out}})} e^{-\epsilon E_0^{\text{out}}(t_f - t_{\text{out}})} \langle 0_{\text{out}} | U(t_{\text{out}}, t_0) \tag{1}$$

and

$$\langle 0_{\text{out}} | U(t_{\text{out}}, t_0) = \langle 0^H_{\text{out}} |, \tag{2}$$

where $\langle 0^H_{\text{out}}|$ is the out-vacuum in the Heisenberg representation.

Exercise 5: Correlation functions in the canonical approach

Calculate the tree-level (connected) two-body scattering amplitude in the theory

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{3!}\phi^3, \tag{3}$$

using Gell-Mann/Low and the LSZ reduction formula. Express your final result in terms of the Mandelstamm variables.

Exercise 6: Invariance under field redefinitions

Let ϕ be a properly normalized field,

$$\langle 0 | \phi(x) | \vec{p} \rangle = \frac{1}{(2\pi)^{3/2}} \frac{e^{ipx}}{\sqrt{2E(\vec{p})}}. \tag{4}$$

Show that the field redefinition $\phi \to \tilde{\phi} = \phi + \delta(\phi)$, where δ is an arbitrary analytic function of ϕ, leaves S-matrix elements invariant.

Exercise 7: $i\epsilon$ prescription

Show that the replacement $t \to t(1 - i\epsilon)$ modifies the propagator of a scalar,

$$\frac{1}{p^2 + m^2} \to \frac{1}{p^2 + m^2 - i\epsilon}. \tag{5}$$

Exercise 8: The generating functional

Show that $Z[J]$ is the generating functional of the n-point correlation functions,

$$\left(\prod_{i=1}^{n} \frac{\delta}{i\delta J^{a_i}(x_i)} \, Z[J] \right) \Bigg|_{J=0} = \frac{\langle 0_{\text{out}} | T\left(\phi_{a_1}(x_1) \cdots \phi_{a_n}(x_n)\right) | 0_{\text{in}} \rangle}{\langle 0_{\text{out}} | 0_{\text{in}} \rangle}. \tag{6}$$

Exercise 9: Ghosts and Tachyons

Consider the following scalar free field Lagrangian,

$$\mathcal{L} = s_1 \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{s_2}{2} m^2 \phi \right), \tag{7}$$

where $s_1 = \pm$ and $s_2 = \pm$ are arbitrary signs. If $s_1 = -1$ we speak of a ghost, and if $s_2 = -1$ we speak of a tachyon.

1. Quantize the theory using the canonical approach for all sign choices.
2. Determine the energy and momentum of the field quanta.
3. Calculate the Feynamn propagator in four-momentum space and determine the required $i\epsilon$ prescription.
4. What type of instabilities do you encounter? In what regime?