**Quantum Field Theory II** Problem Set 3

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Exercise 10: Radiative corrections in $\lambda\phi^4$

Write down (but do not try to calculate explicitly) all the radiative corrections to the propagator to 2nd order in $\lambda$ in the theory

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4. \tag{1}$$

Associate to each correction a Feynman diagram and determine the corresponding symmetry factor.

Exercise 11: The connected generating functional

Express

$$\left.\frac{\delta^4 Z[J]}{\delta\phi(x_1)\cdots\delta\phi(x_4)}\right|_{J=0} \tag{2}$$

in terms of $W[J]$. Interpret your results in terms of diagrams. Which terms correspond to connected graphs?

Exercise 12

This exercise is supposed to illustrate the relation between the generating functional Z and the connected functional W. It is also supposed to show the connection between functional determinants and the calculation of the effective potential (see later problem sets.)

Calculate

$$W \equiv \frac{\int D\phi \exp\left[-\frac{1}{2}\sum_{ab}(D^{-1}_{ab} + M_{ab})\phi_a\phi_b\right]\Big|_{\text{connected}}}{\int D\phi \exp\left[-\frac{1}{2}\sum_{ab} D^{-1}_{ab}\phi_a\phi_b\right]} \tag{3}$$

as an expansion in powers of $M_{ab}$ *to all orders* in $M$. Verify that $W = \log Z$, where

$$Z \equiv \frac{\int D\phi \exp\left[-\frac{1}{2}\sum_{ab}(D^{-1}_{ab} + M_{ab})\phi_a\phi_b\right]}{\int D\phi \exp\left[-\frac{1}{2}\sum_{ab} D^{-1}_{ab}\phi_a\phi_b\right]}. \tag{4}$$

Note that the path integral that defines $W$ runs only over connected diagrams, while the path integral that defines $Z$ contains all diagrams. The value of $Z$ can be calculated directly using the results of Exercise 1,

$$Z = \det{}^{1/2}(1 + DM)^{-1}. \tag{5}$$

Show that this implies that

$$\log Z = -\frac{1}{2}\operatorname{tr}\log(1 + DM) \tag{6}$$

Exercise 13: Self-energy in $\lambda\phi^4$

Calculate the self energy $\Pi^*$ in the theory (1) to first order in $\lambda$. Recall that this is the sum of all 1-PI diagrams with two external lines and outer propagators stripped away. You will find that the self-energy is divergent. Regularize the integral by rotating into Euclidean momentum and imposing a momentum cut-off at $p_E = \Lambda$. How big is the correction to the particle mass?

Exercise 14: Quantum corrections respect linear symmetries

Assume that the action of a theory (and the path integral measure) are invariant under linear transformations,

$$\phi_a(x) \to \int d^4y R_{ab}(x,y)\phi_b(y). \tag{7}$$

Show that the effective action $\Gamma[\phi]$ is also invariant under (7).