Quantum Field Theory II

Problem Set 4

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Exercise 15: The Coleman-Weinberg potential

Use

$$\int d^d p_E f(p_E^2) = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int dp_E \, p_E^{d-1} \, f(p_E^2) \tag{1}$$

to show that the cut-off one-loop contribution to the effective potential of a self-interacting scalar is

$$V_1(\phi) = \frac{\Lambda^2}{32\pi^2} U''(\phi) + \frac{U''(\phi)^2}{64\pi^2}\left[\log \frac{U''(\phi)}{\Lambda^2} - \frac{1}{2}\right] + O(\Lambda^{-2}). \tag{2}$$

What is the structure of the terms of order Λ^{-2}?

Exercise 16: Effective potential as functional determinant

Show that the one-loop contribution to the effective potential can be also derived from a functional determinant (the path integral of a quadratic operator):

$$\exp\left(i\Gamma_1[\bar\phi]\right) = \int D\phi \exp\left[i \int d^4p_1 d^4p_2 \, \frac{\delta^2 S}{\delta\phi(p_1)\delta\phi(p_2)}\bigg|_{\bar\phi} \phi(p_1)\phi(p_2)\right]. \tag{3}$$

Hint: The calculation is similar to the one in Exercise 12.

Note that the integrand on the right hand side is simply $S[\bar\phi + \phi]$ expanded to quadratic order in ϕ.

Exercise 17: Effective potential of two scalars

Calculate the effective potential in the theory

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{\lambda}{4}\phi^2\chi^2. \tag{4}$$

Hint: The second derivative of the potential in Exercise 15 is replaced by a matrix of second field derivatives.