# Quantum Field Theory II

Problem Set 5

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Exercise 18: Effective action in the presence of a background field

Consider the generator of connected correlation functions in the presence of a (not necessarily constant) background field $\bar{\phi}$,

$$\exp\left(iW_{\bar{\phi}}[J]\right) \equiv \int D\phi \exp\left(iS[\bar{\phi}+\phi] + i\int d^4x\, J\,\phi\right) \tag{1}$$

Show that the corresponding effective action

$$\Gamma_{\bar{\phi}}[\phi] \equiv -\int d^4x\, J^{\bar{\phi}}_{\phi}\,\phi + W[J^{\bar{\phi}}_{\phi}] \tag{2}$$

satisfies $\Gamma_{\bar{\phi}}[\phi] = \Gamma[\bar{\phi}+\phi]$, where $\Gamma$ is the conventional effective action.

Exercise 19: Spontaneous breaking of a $U(1)$ symmetry

Consider a complex scalar theory invariant under a $U(1)$ symmetry,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\Phi^*\partial^\mu\Phi + \frac{\mu^2}{2}\Phi^*\Phi - \frac{\lambda}{4}(\Phi^*\Phi)^2, \tag{3}$$

and define

$$\Phi = (F+r)e^{i\theta}, \tag{4}$$

where $F$ is the absolute value of the vacuum expectation value of $\Phi$.

1. Express the Lagrangian in terms of $r$ and $\theta$.
2. Show that $r$ is massive and $\theta$ is massless.
3. How does $\theta$ interact with $r$?
4. What field is the Goldstone boson?

Please turn...

Exercise 20: The linear $\sigma$ model

Consider the Lagrangian of

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{\mu^2}{2} \vec{\phi} \cdot \vec{\phi} - \frac{\lambda}{4} (\vec{\phi} \cdot \vec{\phi})^2, \tag{5}$$

where $\vec{\phi}$ is a vector consisting of $N$ fields, $\vec{\phi} = (\phi_1, \ldots, \phi_N)$. Recall that this model is useful for instance to describe the interactions of pions.

1. What is the symmetry group of the linear $\sigma$ model?
2. How many generators are spontaneously broken?
3. How many Goldstone bosons does the theory contain?
4. What is the unbroken symmetry group?