Quantum Field Theory II

Problem Set 9

Prof. C. Armendariz-Picon

Due Nov. 13, 2008

Exercise 29: Universality of β function coefficients

Show that the first two coefficients of the β function are universal, that is, they do not depend on the precise nature of the renormalization prescription. *Hint:* Assume that renormalized couplings agree at lowest order.

Exercise 30: Low energy effective Lagrangian

Calculate the amplitude for elastic scattering of Goldstone bosons to leading order in E/F using the low-energy effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu\theta\,\partial^\mu\theta + \frac{1}{8\Lambda F^4}(\partial_\mu\theta\,\partial^\mu\theta)^2 + \cdots . \tag{1}$$

Verify that the result is the same you get using the full Lagrangian

$$\mathcal{L} = -\frac{1}{2}\left(1+\frac{r}{F}\right)^2\partial_\mu\theta\,\partial^\mu\theta - \frac{1}{2}\partial_\mu r\,\partial^\mu r - \lambda(2Fr + r^2)^2. \tag{2}$$

Exercise 31: Redundant interactions and field redefinitions

Show that any interaction $\mathcal{O}_i$ proportional to the field equations of motion,

$$\mathcal{O}_i = \int d^4x\, f(\theta, \partial_\mu\theta)\frac{\delta S}{\delta\theta}, \tag{3}$$

can be eliminated by a field redefintion. How would you eliminate an interaction proportional to $\Box\theta$?