Quantum Mechanics I Problem Set 1

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Exercise 1

Let $\vec{v} \in V$ be any element in a vector space V and $\lambda \in \mathcal{K}$ a scalar. Let us furthermore denote by $\vec{0}$ the "identity" in V and by 1 the identity in $\mathcal{K}$.Using exclusively the vector space axioms, prove the following statements.

1. $\vec{0}$ is unique.
2. $\lambda \cdot \vec{0} = \vec{0} \quad \forall \lambda \in \mathcal{K}$.
3. The inverse of $\vec{v}$ is unique.
4. Let us denote the inverse of $\vec{v}$ by $-\vec{v}$. Show that $-\vec{v} = (-1) \cdot \vec{v}$.

Exercise 2

Let $L^2(I)$ be the set of square integrable functions on the real interval I (I could encompass the whole real line.) Show that $L^2(I)$ is a vector space. You will have to define addition and scalar multiplication; use the "natural" ones.

Exercise 3

Suppose that the elements $\{\vec{e}_i\}$ form a basis in a vector space V. Show that the coefficients $\{v_i\}$ in the expansion $\vec{v} = \sum_i v_i \cdot \vec{e}_i$ are unique.

Exercise 4

Show that two vector spaces of the same *finite* dimension are isomorphic.

Exercise 5

Show that the space of all linear operators $A : V \to W$ is a vector space itself.

Exercise 6

Let V a finite dimensional vector space. Show that V and V^* are isomorphic.

Exercise 7

Let V be a finite-dimensional vector space. Find a "natural" isomorphism between V and V^{**}. (This isomorphism is sometimes called the canonical isomorphism.)

Exercise 8

Define the natural addition and scalar multiplication in the tensor product space $V \otimes W$. Your definition should guarantee that $V \otimes W$ is a vector space.