Quantum Mechanics I Problem Set 10

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Exercise 60

Consider scattering off a square potential well.

1. Show that by imposing continuity of ψ and $d\psi/dx$ at the well boundaries we obtain

$$S(E) = \frac{2p\bar{p}}{2p\bar{p}\cos(\frac{2\bar{p}L}{\hbar}) - i(p^2+\bar{p}^2)\sin(\frac{2\bar{p}L}{\hbar})}$$
$$Q(E) = \frac{i(\bar{p}^2-p^2)\sin(\frac{2\bar{p}L}{\hbar})}{2p\bar{p}\cos(\frac{2\bar{p}L}{\hbar}) - i(p^2+\bar{p}^2)\sin(\frac{2\bar{p}L}{\hbar})}.$$

2. Verify that $|S(E)|^2 + |Q(E)|^2 = 1$.
3. Find the poles of $S(E)$. How are they related to the bound states of the well?

Exercise 61

Using Schrödinger's equation, show that the probability density and current satisfy the "conservation equation"

$$\frac{\partial P}{\partial t} = -\vec{\nabla}\cdot\vec{j}.$$

Exercise 62

Imagine that a state vector (a resonance) has wave function (in the energy representation)

$$\langle E|\psi\rangle = \sqrt{\frac{\Gamma}{2\pi}}\frac{1}{E - E_0 + i\Gamma/2}.$$

1. What is the origin of the factor $\sqrt{\Gamma/(2\pi)}$?
2. Compute how a state with this wave function evolves in time, that is, compute $\langle\varphi|U(t,t_0)|\psi\rangle$, where $|\varphi\rangle$ is an arbitrary, time-independent state.
3. Interpret your results. What is the relation between Γ and the "lifetime" of the state.

Please turn for more...

Exercise 63

Consider a Gaussian wave packet approaching a potential step.

1. Find its time evolution.
2. Construct a movie of its motion by drawing snapshots of the wave packet at different times.
3. Interpret your results

Hint: Use all your powers. If you get stuck, look for guidance in a textbook.

Exercise 64

Consider an infinitely deep weel, $V_0 = -\infty$. Find the energy eigenstates and eigenvalues of even and odd parity bound states. What are the values of the wave function at the walls? Draw the first four ave functions. Verify that the "haphazard" collection of theorems I mentioned in class is satisfied.

Bonus Exercise 65

Consider the delta-function potential $V(x) = V_0\,\delta(x)$, with $V_0 < 0$.

1. Determine the energies of the bound states and the corresponding wave functions. Do they have to be continuous? How about their derivatives?
2. Consider now scattering off the delta function potential. Calculate the transmission and reflection amplitudes. Find the resonances of the potential and the poles of the transmission amplitude in the complex energy plane. Interpret your results.

Optional Holiday Exercise 66

Consider the potential

$$V(x) = \begin{cases} 0, & |x| > L + w \\ V_1, & L \leq |x| \leq L + w \\ V_0, & |x| \leq L \end{cases} ,$$

where $V_1 > 0$ and $V_0 < 0$.

1. Compute the transmission amplitude $S(E)$.
2. Find the bound states and resonances.
3. How does the width of the resonances depend on the parameters of the potential?