Quantum Mechanics I

Problem Set 11

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Exercise 67

In class, we defined an operator $\hat{a}$ as a particular combination of $\hat{x}$ and $\hat{p}$. Show that the Hamiltonian of an harmonic oscillator is

$$\hat{H} = \hbar\omega \left(\hat{N} + \frac{1}{2}\right),$$

where $\hat{N} = \hat{a}^\dagger \hat{a}$ is the number operator.

Exercise 68

Show that

$$[\hat{a}, \hat{a}^\dagger] = 1,$$
$$[\hat{N}, \hat{a}] = -\hat{a},$$
$$[\hat{N}, \hat{a}^\dagger] = \hat{a}^\dagger.$$

How can we interpret each of these relations?

Exercise 69

A "tachyonic" oscillator is an harmonic oscillator with imaginary frequency (the harmonic potential is upside-down.)

1. Calculate the classical trajectories of the tachyonic oscillator
2. Find the spectrum of the oscillator
3. Imagine that at time t_0 the wave function of the oscillator is a Gaussian centered around the top of the potential ($x = 0$.) Find the wave function of the system at time $t > t_0$.
4. Is the time evolution of the tachyonic oscillator unitary?

Exercise 70

Show that the coherent state $|\alpha$ can be expressed as

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \exp\left(\alpha\, \hat{a}^\dagger\right) |0\rangle.$$

Coherent state contain an infinite number of excitations.

Please turn...

Exercise 71

Show that for a coherent state

$$\Delta x = \sqrt{\frac{\hbar}{2m\omega}}, \quad \Delta p = \sqrt{\frac{m\hbar\omega}{2}}.$$

Coherent states are states of minimal uncertainty.

Exercise 72

Show that

$$U(t,0)|\alpha\rangle = e^{-i\omega t}\,|\alpha\, e^{-i\omega t}\rangle.$$

Coherent states remain coherent.

Exercise 73

Read the article

- R. J. Glauber, "Coherent and Incoherent States of the Radiation Field," Phys. Rev. **131**, 2766 (1963).

This is an article that illustrates many of the concepts, formalism and ideas that we have discussed in class. Make sure that you go through Sections I-VIII, and try to connect everything the author mentions with the course.

1. Try to draw an analogy between the operators we have dealt with in class (e.g. $\hat{Q}$ and $\hat{P}$) and the (field) operators discussed by Roy Glauber in Section II. In particular, study his equations (2.10)-(2.23). How do they resemble the ones of an harmonic oscillator? What's the difference?
2. Sections III-VIII deal with coherent states of a single harmonic oscillator. Make sure that you understand what Glauber presents here. Summarize the most important properties of coherent states discussed throughout these sections.
3. The author makes a reference to the Baker-Campbell-Hausdorff equation. How does it differ from what I told you in class?
4. You can skip sections IX and X.
5. What are coherent states useful for?