Prof. C. Armendariz-Picon Due Sept. 22, 2006

Exercise 9

Prove the Cauchy-Schwarz inequality, $|(\vec{u}, \vec{v})| \leq \|\vec{u}\| \cdot \|\vec{v}\|$.

Exercise 10

Assume that V is a finite dimensional vector space over $\mathbb{R}$. Show that the dual map, $\vec{v} \mapsto \vec{v}^{\,*}$ defined in class is an isomorphism between V and V^*. Show that in general

$$(\vec{v} + \lambda\, \vec{w})^* = \vec{v}^{\,*} + \lambda^* \, \vec{w}^{\,*}.$$

Exercise 11

Show that

i) $A^\dagger$ is linear

ii) $(A + \lambda\, B)^\dagger = A^\dagger + \lambda^* B^\dagger$

iii) $A^{\dagger\dagger} = A$

iv) $(A\, B)^\dagger = B^\dagger \, A^\dagger$.

Exercise 12

Show that the operators on $L^2(\mathbb{R})$

$$\begin{aligned} \hat{x} : f(x) &\mapsto x \cdot f(x) \\ \hat{p} : f(x) &\mapsto -i \frac{df}{dx} \end{aligned}$$

are self-adjoint. What changes if we consider a closed interval $I = [a, b]$ instead of $\mathbb{R}$?

Exercise 13

Show that $(\vec{v}, A^\dagger A\, \vec{v}) \geq 0$ ($A^\dagger A$ is positive.)

Exercise 14

Show that the eigenvalues of hermitean operators are real.

Please turn for more exciting exercises!

Exercise 15

Show that eigenvectors of hermitean operatosr to different eigenvalues are orthogonal.

Exercise 16

Let U be a unitary operator. Prove that

i) $U^\dagger U = UU^\dagger = \mathbf{1}$

ii) The eigenvalues of unitary operators have unit norm

iii) $\|U\vec{v}\| = \|\vec{v}\|$.