Quantum Mechanics I Problem Set 3

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Exercise 17

Show that $\sum |i\rangle\langle i| = \mathbf{1}$.

Exercise 18

Prove that

i) The dual of $A|\psi\rangle$ is $\langle\psi|A^\dagger$

ii) If $A|\psi\rangle = \lambda|\psi\rangle$ ($|\psi\rangle$ is an eigenvector of A with eigenvalue λ), $\langle\psi|A^\dagger = \lambda^*\langle\psi|$.

iii) The matrix elements of an operator satisfy $(A^\dagger)_{ij} = (A_{ji})^*$.

Exercise 19

Show that the matrix $\Lambda_{\alpha i}$ defined in class is unitary.

Exercise 20

Using Dirac's notation, show that $(A \cdot B)_{ij} = \sum_k A_{ik}B_{kj}$.

Exercise 21

Show that the images of a set of basis vectors uniquely determine a linear operator.

Exercise 22

Solve the exercise given in class.

Exercise 23

Suppose that a spin 1/2 particle is in the state

$$|\psi\rangle = \frac{1}{\sqrt{|\alpha|^2 + |\beta|^2}} \left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right).$$

What is the probability of observing the spin of the particle to be up? What is the origin of the factor $1/\sqrt{|\alpha|^2 + |\beta|^2}$?

Please turn for more challenging exercises!

Exercise 24

Let A be the operator associated with the observable $\mathcal{A}$. Show that the probability of obtaining any result upon measurement of $\mathcal{A}$ is one.

Exersice 25

What happens to the probabilities of measurement outcomes if a state vector is multiplied by a (constant) phase?

$$|\psi\rangle \to e^{i\varphi}|\psi\rangle$$