Quantum Mechanics I

Problem Set 6

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Due Oct 20, 2006

Exercise 36

Show that the operators $\hat{x}^i$ and $\hat{p}_j$ that we defined in class in the context of spatial translations satisfy the commutation relation $[\hat{x}^i, \hat{p}_j] = \delta^i{}_j$.

Exercise 37

Show that in momentum representation

$$(\hat{x}^i\psi)(\vec{p}) = i\hbar\frac{d\psi}{dp_i}.$$

Exercise 38

Prove that $|\langle\psi|\phi\rangle| \geq \frac{1}{2}|i\langle\psi|\phi\rangle - i\langle\phi|\psi\rangle|$.

Exercise 39

Show that $[\hat{x} - \langle\hat{x}\rangle, \hat{p} - \langle\hat{p}\rangle] = [\hat{x}, \hat{p}]$.

Exercise 40

Show that the solution of the equation

$$\frac{|\psi_x\rangle}{\sqrt{\langle\psi_x|\psi_x\rangle}} = \frac{\psi_p\rangle}{\sqrt{\langle\psi_p|\psi_p\rangle}},$$

where $|\psi_x\rangle$ and $|\psi_p\rangle$ are the states we defined in class is

$$\langle x|\psi\rangle = \frac{1}{(2\pi)^{1/4}(\Delta x)^{1/2}}\exp\left[\frac{i\langle p\rangle x}{\hbar} - \frac{(x-\langle x\rangle)^2}{4(\Delta x)^2}\right].$$

What is the corresponding wave function in momentum representation? Show that $\langle\hat{p}\rangle = \langle p\rangle$, $\langle\hat{x}\rangle = \langle x\rangle$, $\langle\Delta\hat{x}\rangle = \langle\Delta x\rangle$, $\langle\Delta\hat{p}\rangle = \frac{\hbar}{2\Delta x}$. In these equations, the left hand side is the expectation value of an operator, and the right hand side contains the parameter that appears in the wave function above.

Exercise 41

Read Chapter IV.II, Section 10, in Messiah's "Quantum Mechanics."