Quantum Mechanics I Problem Set 7

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Exercise 42

Read the hand-out of E. P. Wigner's, "Group Theory and Its Applications to the Quantum Mechanics of Atomic Spectra."

(Note: Probably I'll include a question on any of the readings in the midterm exam.)

Exercise 43

Show that in the Heisenberg representation

$$\frac{d\hat{A}}{dt} = \frac{1}{i\hbar}[\hat{A}, \hat{H}] + \frac{\partial \hat{A}}{\partial t}$$

Exercise 44

Assume that the operators $\hat{q}$ and $\hat{p}$ satisfy the canonical commutation relations $[\hat{q}, \hat{p}] = i\hbar$. Show that

$$[\hat{q}, \hat{p}^n] = i\hbar\, n\, \hat{p}^{n-1}, \qquad [\hat{p}, \hat{p}^n] = 0,$$
$$[\hat{p}, \hat{q}^n] = -i\hbar\, n\, \hat{q}^{n-1}, \qquad [\hat{q}, \hat{q}^n] = 0.$$

Show that these relations imply the formal equations

$$[\hat{q}, \hat{A}(\hat{q}, \hat{p})] = i\hbar \frac{\partial \hat{A}}{\partial \hat{q}},$$
$$[\hat{p}, \hat{A}(\hat{q}, \hat{p})] = -i\hbar \frac{\partial \hat{A}}{\partial \hat{p}}.$$

Exercise 45

Show that the commutator is bilinear

$$[A + \lambda B, C] = [A, C] + \lambda [B, C],$$
$$[A, B + \lambda C] = [A, B] + \lambda [A, C],$$

and satisfies the chain rule

$$[AB, C] = A[B, C] + [A, C]B.$$

Please turn for more ...

Problem 46

Consider the Lagrangian of a particle in a homogeneous magnetic field pointing along the z-direction.

1. What are the canonical momenta of the particle?
2. Find the Hamiltonian of the system.
3. Quantize the theory by imposing canonical quantization relations.
4. What are the velocity operators?
5. Compute the commutators of the velocity operators with themselves.