Quantum Mechanics I — Problem Set 8

Prof. C. Armendariz-Picon — Due Nov 9, 2006

Exercise 47

Show that upon measurement of A, $\rho \to \sum_a P_a \rho P_a$, where P_a is the projection operator onto the eigenspace of the eigenvalue a, and ρ is the density matrix.

Exercise 48

Read David Mermin's "Is the moon there when nobody looks?"

Exercise 49

Read H. Everett's "Relative State Formulation of Quantum Mechanics."

Exercise 50

Show that

$$\hat{p}^2 \psi(x) = -\hbar^2 \frac{d^2\psi}{dx^2}.$$

Exercise 51

Find the eigenfunctions of the free Hamiltonian of a particle in n dimensions.

Exercise 52

Verify that

$$U(t, x; t_0, x_0) = \int \frac{dp}{2\pi\hbar} \exp\left[\frac{i}{\hbar} p(x - x_0)\right] \exp\left[-\frac{i}{\hbar} E(p)(t - t_0))\right]$$

is the time propagator in the position representation, $U(t, x; t_0, x_0) = \langle x | U(t, t_0) | x_0 \rangle$.

Exercise 53

Show that

$$U(t, x; t_0, x_0) = \frac{m}{2\pi i(t - t_0)} \exp\left(\frac{im(x - x_0)^2}{2\hbar(t - t_0)}\right),$$

and verify that $U(t, x; t_0, x_0)$ is proportional to $\exp(iS_{\text{cl}}/\hbar)$, where S_{cl} is the action along the classical trajectory joining (t_0, x_0) and (t, x).

Please turn for more ...

Exercise 54

Prove the following extremely important equation,

$$\int d^n x \exp\left[-\frac{1}{2}\vec{x}\cdot A\vec{x} + \vec{J}\cdot\vec{x}\right] = \left(\frac{(2\pi)^n}{\det A}\right)^{1/2} \exp\left(\frac{1}{2}\vec{J}\cdot A^{-1}\vec{J}\right).$$

Verify that the integral is proportional to the exponential evaluated at the stationary point of the integrand.