Quantum Mechanics I Problem Set 9

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Exam Exercise Reloaded

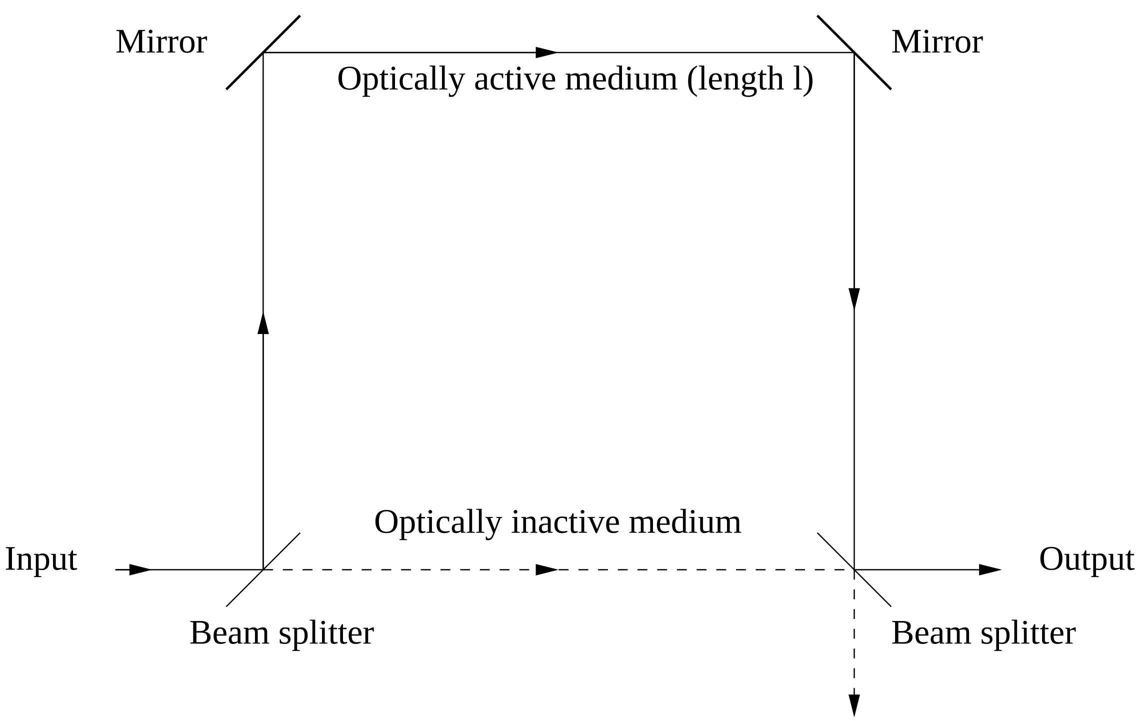
We shine a beam of right-polarized photons into a symmetrical beam splitter. The beam splitter transmits vertically polarized light and reflects horizontally polarized light by 90 degrees. The different polarization states are related by the equations

$$
\begin{aligned}
|R\rangle &= \frac{1}{\sqrt{2}}\left(|V\rangle + |H\rangle\right), \\
|L\rangle &= \frac{1}{\sqrt{2}}\left(|V\rangle - |H\rangle\right),
\end{aligned}
$$

where the states $|L\rangle$, $|R\rangle$, $|V\rangle$ and $|H\rangle$ denote, respectively, left, right, vertically and horizontally polarized light.

The reflected beam passes through an actively active medium, where the Hamiltonian is given by $H_{\text{active}} = \alpha \cdot (|R\rangle\langle R| - |L\rangle\langle L|)$, and is conducted by a series of mirrors that do not affect its polarization to a second identical beam splitter, where the two beams merge in the output. The length of the optically active portion of the interferometer is l.

1. What is the probability to measure a left-polarized photon in the output?
2. Imagine we determine (e.g. by measuring the beam splitter recoil) which way photons are taking. What is the probability to detect a photon in the output?



Exercise 55

Calculate the propagator in momentum representation, $\langle p|U(t,t_0)|p_0\rangle$, and interpret your result.

Exercise 56

Consider the initial Gaussian wave packet

$$\psi(t_0,x) = \frac{1}{(2\pi)^{1/4}(\Delta x_0)^{1/2}} \exp\left(\frac{ip_0 x}{\hbar} - \frac{(x-x_0)^2}{4\Delta x_0^2}\right).$$

Compute the wave function at time t. What is the expectation value of the position operator at time t? What is the position uncertainty Δx at time t? If you get stuck, find the answer in a book. Verify that the position uncertainty grows with time.

Imagine now that a body of 1 kg mass has an initial velocity uncertainty of 10^{-7} m/sec. What is its initial position uncertainty? How long does it take for this uncertainty to double?

Exercise 57

Let P be the parity operator. Show that

$$P^{-1}\hat{p}P^{=} - \hat{p}, \quad P^{-1}V(\hat{x})P = V(-\hat{x}).$$

Exercise 58

Show that the conditions

$$\begin{aligned} \beta &= \alpha \tan(\alpha L) \quad (even) \quad \text{and} \\ \beta &= -\alpha \cot(\alpha L) \quad (odd), \end{aligned}$$

are equivalent to

$$\begin{aligned} |\cos\alpha L| &= \frac{\alpha}{\frac{\sqrt{2m|V_0|}}{\hbar}}, \tan(\alpha L) > 0 \quad (even) \quad \text{and} \\ |\sin\alpha L| &= \frac{\alpha}{\frac{\sqrt{2m|V_0|}}{\hbar}}, \cot(\alpha L) > 0 \quad (odd). \end{aligned}$$

Exercise 59

Consider an infinitely deep weel, $V_0 = -\infty$. Find the energy eigenstates and eigenvalues of even and odd parity bound states. What are the values of the wave function at the walls? Draw the first four ave functions. Verify that the "haphazard" collection of theorems I mentioned in class is satisfied.